2022

Optimizing Pension Outcomes Using Target Volatility Investment Concept

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Zefeng Bai

A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Ph.D. in Business

2022

Program Authorized to Offer Degree:
Department of Mathematical Sciences

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Dedication

This dissertation is dedicated to my wife, Yuchen Liu.

Acknowledgments

First and foremost, I would like to express my deepest gratitude to Victoria Steblovskaya, who serves as my advisor and PhD committee chair, for her enormous support and encouragement throughout my PhD study. She has taught me not only the research and the teaching skills required to complete the program, but also the expertise to work as a professional scholar, researcher, and educator. My completion of this PhD program and my current growth in professional career could not be possible without her.

I would like to further extend my gratitude to my PhD committee members: Dessislava Pachamanova, Yicheng Kang, and Jahangir Sultan for bringing new perspectives, knowledge, and guidelines to my research work. Their valuable comments and suggestions are the key to the completion of this dissertation study.

My sincere appreciation goes to Kai Wallbaum, for his professional mentorship during my internship in Germany as well as his continuous guidance on my research project thereafter. His insightful advice from the perspective of a pension practitioner has built a bridge that connects my research work with real-life, industrial application.

I would like to say a huge thank you to the Department of Mathematical Sciences as well as the PhD program at Bentley University, which jointly created an active, involved, and diversified learning environment. I would like to especially thank Lucy Kimball, Mingfei Li, and Dominique Haughton for your time and efforts on strengthening my analytical and teaching skills. I'm also particularly thankful to Jay Thibodeau, Martin Conyon, and Patricia Caffrey for continuously developing an attractive PhD program where students and faculty members with a diversified skill set can collaborate and thrive.

I'm very grateful to have the chance to meet my PhD colleagues: Cristina Alberti, Joanna Walton, Jooanne Choi, Landi Morries, Nitin Jain, Olga Biedova, Roman Kurdyukov, Tao

Li, Tyler Wasson, Ying Wang, and Zheng(Lisa) Liu. Having the pleasure to work with you made the PhD journey more enjoyable and interesting than I would expect.

Finally, I would like to extend my appreciation to my wife, Yuchen Liu and other family members and close friends for their selfless encouragement and support while I am living and studying in the United States.

Abstract

Optimizing Pension Outcomes Using Target Volatility Investment Concept

Zefeng Bai

Chair of the Supervisory Committee:

Professor Victoria Steblovskaya

Department of Mathematical Sciences

The target volatility strategy is a very popular investment concept in financial marketplace. For my dissertation, I focus on studying the target volatility investment concept in application to pension accumulation as well as decumulation stages. Additionally, I extend a basic target volatility strategy by introducing trading boundaries to its asset allocation mechanism. My dissertation study follows a three-paper format.

In paper one, we propose a new pension strategy that aims at improving the protection of a long-term pension plan in volatile market conditions. Over a hypothetical twenty-year pension scheme, we show that our newly proposed strategy, which attaches a target volatility mechanism to a lifecycle strategy, could provide more effective capital protection and risk control for pension investment vehicles. In addition, we show that our proposed strategy has an improved portfolio diversification effect and market timing skills compared to a benchmark pension strategy. Our results are robust with a consideration of transaction costs.

In paper two, we enhance the retirement coverage of several conventional retirement plans by using a target volatility strategy with interest rate dependent target volatility levels. Using the Monte Carlo simulation approach, we find that the retirement portfolio enhanced by the target volatility mechanism shows a significantly higher level of confidence to achieve

required income levels compared to the conventional retirement portfolio. Therefore, the target volatility investment strategy could be a suitable alternative for investors who look for a higher level of stability in retirement coverage.

In paper three, we attempt to reduce the transaction costs of a target volatility portfolio by adding market risk calibrated rebalancing boundaries to its asset allocation mechanism. A constraint optimization problem based on investor-relevant optimization criteria is formulated. A numerical optimization algorithm to find an optimal rebalancing boundary level is presented. Illustrative numerical results within the Black-Scholes environment, as well as using real market data are reported. The comparative analysis on different market scenarios suggests that the target volatility portfolio with rebalancing boundaries can effectively reduce portfolio transaction costs and improve portfolio returns. Our findings have important applications given the popularity of the target volatility investment strategies among financial practitioners.

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Introduction

The target volatility investment concept is a dynamic asset management approach, which aims at controlling portfolio risk in the volatile market conditions. Due to the 2008 financial crisis and the recent market turbulence triggered by the COVID-19 pandemic, we observe an increasing popularity of the target volatility concept adopted in various pension plans and long-term investment funds. Thus, for my dissertation, I focused on studying the target volatility investment concept in application to pension accumulation as well as decumulation stages. Additionally, I extended a basic target volatility strategy by introducing trading boundaries to the volatility target asset allocation mechanism. This will lead to reducing transaction costs encountered in the process of rebalancing a target volatility portfolio. The goal of my dissertation is as follows:

- Improving the protection of a long-term pension plan, such as the lifecycle pension strategy, using the target volatility investment concept in pension accumulation stage.
- Enhancing retirement coverage of conventional retirement portfolios using the target volatility investment concept in pension decumulation stage.
- Reducing the deleterious effect of transaction costs on portfolio returns by adding the rebalancing boundaries to the volatility target asset allocation mechanism.

The three objectives mentioned above are addressed in Chapter 2, Chapter 3, and Chapter 4 of this dissertation.

For the first objective, we propose a new pension strategy which attaches the target volatility investment concept to an existing lifecycle pension strategy. After computing different numerical measurements for portfolio performance (i.e., risk-return ratio, annualized volatility, value-at-risk, conditional value-at-risk, and downside deviation), we use the Jensen's alpha regression and Henriksson and Merton measure to examine the diversification effect and market timing skill of pension portfolio. Our results suggest that the lifecycle strategy attached with a target volatility mechanism can provide a more effective asset protection in volatile market conditions.

For the second objective, we use the target volatility strategy to enhance the retirement coverage of several conventional retirement portfolios in pension decumulation stage. To do so, we first consider a dynamic target volatility strategy with the target volatility level dependent on market interest rates and then use the strategy as an extra layer of portfolio protection. Using the Monte Carlo simulation approach, we evaluate the performance of the dynamic target volatility strategy under different interest rate environments. Our simulation results show that the dynamic target volatility mechanism can further improve the sustainability of a conventional retirement portfolio in pension decumulation stage. Thus, we conclude that the target volatility investment strategy could be a suitable alternative for investors who look for a higher level of stability in retirement coverage after they retired.

For the third objective, we attempt to reduce the transaction costs of a target volatility portfolio by adding market risk calibrated rebalancing boundaries to its asset allocation mechanism. We formulate a constraint optimization problem based on investor-relevant optimization criteria and carry out a numerical optimization algorithm to find an optimal rebalancing boundary level. Illustrative numerical results within the Black-Scholes environment, as well as using real market data are reported. The comparative analysis on different market scenarios suggests that the target volatility portfolio with rebalancing boundaries can effectively reduce portfolio transaction costs and improve portfolio returns. Our findings have important applications given the popularity of the target volatility investment strategies among financial practitioners.

This dissertation study contributes to the literature in the field of pension management.

First, the dissertation study extends the existing literature about adopting the target volatility strategy in pension management domain. It shows that better pension outcomes can be achieved by combining the target volatility investment concept with an existing pension management strategy such as the lifecycle pension strategy. It also shows that the risk control mechanism embedded in the target volatility strategy can provide an effective asset protection when a pension portfolio encounters turbulent market events.

Second, given the limited discussion on implementing the target volatility strategy in pension decumulation stage, we propose using the target volatility strategy as a continuous measure to enhance the retirement coverage of retirement portfolios after people retired. We show that the target volatility investment concept can not only improve various pension outcomes in the pension accumulation stage but also serve as a useful ”patch” to the conventional retirement portfolios and allow them to provide a more sustainable retirement coverage in pension decumulation stage. This is reflected in an enhanced portfolio survival rate and length of retirement coverage.

Third, the proposed dissertation study contributes to the literature studying transaction costs in asset management and long-term investment strategies. In asset management, the presence of transaction cost has been a major factor affecting portfolio returns. Mitigating the deleterious effect of transaction cost could be an effective approach to improve the performance of an asset management strategy. Thus, we believe our findings will be of interest to practitioners in portfolio management field.

Optimizing Pension Outcomes Using Target-Driven Investment Strategies: Evidence From Three Asian Countries With The Highest Old-Age Dependency Ratio[1]

2.1 Introduction

How to optimize pension outcomes for pension plans in volatile markets? This is a question that has been debated, internationally, by both scholars and practitioners. Over the past several decades, various investment strategies have garnered significant attention in pension discussions. For example, different time-series momentum strategies (Baltas and Kosowski, 2013), risk parity approaches (Kazemi, 2012), and target-date pension investment strategies have become important in financial investment practice due to their suitable asset allocation concepts in response to rapidly changing market conditions (Forsyth and Vetzal, 2019). Under most target-date pension concepts, investors benefit from the embedded asset allocation process, which reduces risky assets when the retirement date nears. Prior research has shown that a simple target-date pension strategy often outperforms other investment strategies and can generate better pension outcomes in various market scenarios (Bodie and Treussard, 2007; Elton et al., 2015). As one of the most commonly used target-date pension strategies, the lifecycle pension management strategy has become an important pillar of old-age provision in the United States and Europe. In the foreseeable

[1] This chapter is published in Bai, Z., & Wallbaum, K. (2020). Optimizing Pension Outcomes Using Target-Driven Investment Strategies: Evidence from Three Asian Countries with the Highest Old-Age Dependency Ratio. Asia-Pacific Journal of Financial Studies, 49(4), 652-682. ©2020, Korean Securities Association. DOI: https://doi.org/10.1111/ajfs.12310.

future, the lifecycle concept will also play a more critical role in other global pension markets.

Motivated by the most recent global financial market drawdown caused by the COVID-19 pandemic (Baker et al., 2020) and the low expected market return environment (Horneff et al., 2018), we propose a new combined pension management strategy that aims to improve the traditional lifecycle pension investment concept in different market conditions. To develop this new strategy, we attach a dynamic risk management overlay called the target volatility mechanism to a simple lifecycle pension strategy as an additional investment protection mechanism for pension portfolios in more volatile markets. We set up a 20-year hypothetical pension scheme and conduct corresponding analyses based on historical time series to compare the performances of two pension strategies—the simple linear lifecycle pension strategy (hereafter referred to as the lifecycle strategy) and the simple linear lifecycle pension strategy with the target volatility overlay (hereafter referred to as the target volatility strategy or the combined strategy) in three investment regions: Japan, China, and South Korea. We focus on these three countries because they are projected to have the highest old-age dependency ratio by 2050 (United Nation, 2020). We believe that proposing a suitable pension investment strategy for them could help reduce the financial burden placed on their working forces as a result of supporting the old-age population in the future. Our numerical results suggest that the target volatility strategy could lead to higher portfolio returns in these three Asian markets. More importantly, we also find that the target volatility strategy could provide effective capital protection when a pension portfolio encounters market turbulence, such as the 2008 financial crisis, and that it could reduce the overall portfolio downside risk over the hypothetical pension scheme. Consequently, we find that the target volatility mechanism could improve various pension outcomes and offer effective capital protection when combined with the lifecycle pension strategy.

2.2 Literature Review

For decades, one of the most widely used pension strategies has been the lifecycle pension strategy, which belongs to the target date concept family. The lifecycle pension strategy reduces the risky asset exposure of a pension portfolio as it approaches the "target date" (Graf, 2017) so that the pension portfolio is secure as a future retiree approaches retirement age. Benefits of the lifecycle pension strategy have been well documented. Prior studies have shown that pension investors benefit from the lifecycle pension strategy because of its simple asset management procedure that is based on a fixed glide path (Tang and Lin, 2015). Hence, the lifecycle pension strategy is often considered to be the default pension investment strategy in pension practice. Other studies have shown that the lifecycle pension strategy is a flexible portfolio management strategy that could be combined with other pension concepts in order to achieve different pension targets (Mitchell et al., 2009). However, recent studies have cast new doubt on the classical lifecycle pension investment strategy. For instance, although lifecycle pension strategies have been shown to be superior to money market funds, they often underperform the balanced mandates that hold a constant asset allocation of 50% risky assets and 50% risk-free assets throughout the pension accumulation phase (Blanchett and Kaplan, 2018). Further studies have shown that the traditional lifecycle pension strategy needs a more dynamic risk management concept, which could react better to unforeseeable market turbulence and adjust asset allocation accordingly (Dhillon et al., 2016).

In response to the current low market return environment and volatile equity market conditions, which have brought new challenges to existing pension investment strategies in different markets, prior studies have shown the advantages of implementing various dynamic risk control measures in order to improve the performance of existing long-term investment strategies. For example, efficient volatility estimations in time-series momentum strategies could significantly reduce portfolio turnover without deterring their performance (Baltas and Kosowski, 2020) and a dedicated risk management method for the risk of momentum could improve the performance of a momentum strategy (Barroso and Santa-Clara, 2015). Additional studies have shown that a dynamic momentum strategy that uses forecasted return and variance of momentum could outperform a static one (Daniel and Moskowitz, 2016) and that a diversified portfolio of time-series momentum strategies often yields better performance in extreme market conditions (Moskowitz et al., 2012). The target volatility mechanism could represent another important dynamic pension investment strategy embedded with a risk control component (Albeverio et al., 2013; Cirelli et al., 2017). In contrast to the traditional lifecycle pension strategy, which follows a glide-path asset allocation concept, the target volatility mechanism adjusts pension portfolio allocation on the basis of pre-specified target volatility and realized market volatility (Albeverio et al., 2018). In other words, the target volatility mechanism allows for a flexible portfolio asset adjustment between risky assets and risk-free assets, regardless of the pension target date. Moreover, the target volatility mechanism is found to be an effective capital measure for different assets over a long investment timeframe (Albeverio et al., 2019). Furthermore, it is well acknowledged that pension management strategies with a target risk management component might be more suitable for pension investors in comparison to traditional target-date pension strategies in volatile markets (Elton et al., 2016; Moreira and Muir, 2017; Barro et al., 2019).

Owing to the limitations of the lifecycle pension strategies identified in previous studies and possibly unforeseeable market turbulence in the future, the present study proposes a new pension investment strategy that uses the target volatility mechanism as a "patch" for the classical linear lifecycle pension strategy. The contribution of this paper is twofold. First, our study contributes to the existing literature on pension management strategies by showing that the target volatility mechanism can improve pension performance when it is combined with the commonly used lifecycle concept in the markets examined in this paper. To the best of our knowledge, this is also the first attempt to examine the target volatility mechanism in three countries (i.e. Japan, South Korea, and China) that are projected to have the highest old-age dependency ratio. Second, this study suggests that the target volatility mechanism embedded with a daily rebalancing asset management concept is feasible for practical implementation even when taking transaction costs into consideration. Subsequently, our paper presents guidelines for both scholars and practitioners on how to implement the lifecycle concept in an alternative manner.

2.3 Two Pension Strategies

Pension investment strategies often aim to provide a sufficient amount of capital for retirees by the time they reach retirement age so that their living expenses can be covered after they retire. To optimize different pension portfolio outcomes, both pension practitioners and scholars have proposed various asset management approaches. For instance, the balanced mandate, which consists of 50% equity and 50% bond, is often shown to be a simple and effective investment strategy in a pension accumulation context (Finke and Blanchett, 2016). Alternatively, the lifecycle pension strategy is often selected as the default portfolio management strategy for the 401K in the United States (Mitchell and Utkus, 2012). This section introduces the lifecycle and target volatility strategies as well as our new setups for them.

2.3.1 Lifecycle strategy

In contrast with other pension investment strategies—such as the balanced mandates, which maintain a constant asset allocation of 50% risky and 50% risk-free assets over the pension accumulation phase—the lifecycle strategy gradually reduces risky assets that are embedded in the pension portfolio as its target date approaches (i.e. usually the investor's retirement date). Specifically, a pension portfolio under a lifecycle strategy often consists of a large portion of risky assets (e.g. 100% pure equities) at the beginning of the pension accumulation stage in order to enhance its probability to gain from risky markets. As the investor approaches retirement age, the asset allocation of their pension portfolio gradually leans more toward risk-free assets, such as bonds or money market instruments, in order to maximize pension portfolio protection.

2.3.2 Target volatility strategy

To date, studies have shown that different target-driven investment strategies, such as the target volatility mechanism, could provide effective capital protection under various volatile market conditions (Blake et al., 2013; Albeverio et al., 2018). In contrast to the lifecycle strategy, which consistently decreases the risky asset allocation over the pension accumulation phase, the target volatility strategy contains a dynamic risk management component that aims to adjust the asset allocation of a pension portfolio on a daily, monthly, or yearly basis. This has particular advantages because pension portfolios that are under the lifecycle strategy usually miss the opportunity to gain from risky markets as they get closer to their target date. Furthermore, the target volatility strategy is also a performance-driven investment strategy that could change the asset allocation of a pension portfolio based on market performance. Hence, a pension portfolio under the target volatility strategy could react better to rapidly changing market conditions.

To implement the target volatility strategy, a pre-determined target volatility is first specified at the beginning of the pension accumulation stage. Then, the asset allocation of the pension portfolio is further determined using a ratio between the target volatility and the realized market volatility, as follows:

$$\alpha_t = min\{\frac{T}{V_t}; L\} \tag{2.1}$$

$$\beta_t = 1 - \alpha_t \tag{2.2}$$

In the above formulas, T represents the target volatility specified at the beginning of the pension accumulation stage, V_t stands for the realized market volatility derived from daily market returns, and L represents the maximum risky asset allocation that prevents the portfolio from excessive exposure to risky assets. Subsequently, α_t and β_t represent the risky and risk-free asset allocation, respectively.

Next, we provide an example of a pension portfolio under a target volatility strategy. Let us assume that a pension investor chooses a target volatility strategy with a target volatility of 20% and that the realized market volatility was 40%. The maximum risky asset allocation would be capped at 150%.

$$\alpha_1 = min\{\frac{20\%}{40\%};150\%\} = 50\%$$
$$\beta_1 = 1 - 50\% = 50\%$$

The investor would allocate 50% of the capital into risky assets and the remaining 50% into risk-free assets. In the next month, let us suppose that the realized market volatility decreases to 15%.

$$\alpha_1 = min\{\frac{20\%}{15\%};150\%\} = 133\%$$
$$\beta_1 = 1 - 133\% = -33\%$$

Then, the investor would allocate 100% of the capital into risky assets and would borrow an additional 33% of the capital from local banks to invest in risky assets.

2.3.3 Pension strategy setups

For the lifecycle strategy, we consider a simple pension portfolio that consists of a risky asset portion represented by equities and a risk-free asset portion represented by bonds. The pension portfolio starts at 100% equity and 0% bond at the beginning of the 20-year pension scheme, switching 5% of the equities to bonds each year. The idea is to secure the pension portfolio as the end of the 20-year pension scheme approaches (i.e. the target date).

A pension portfolio under the target volatility strategy also consists of both risky and risk-free asset portions. However, the risky asset portion of this pension portfolio under the target volatility strategy consists of a combination of equities and bonds, which follows the lifecycle strategy. The risk-free asset portion of the pension portfolio under the target volatility strategy includes money market instruments, which are represented by local three-month internal bank rates or immediate bank rates. Then, we use the target volatility mechanism to determine the risky and risk-free asset allocation of the pension portfolio under the target volatility strategy. To remain consistent with the concept of the lifecycle pension strategy, which gradually reduces the risky asset portion of a pension portfolio as it approaches the target date, we also set up the target volatility strategy to have a constant target volatility decreasing rate (hereafter referred to as a target volatility scenario). The idea is to secure the pension portfolio by lowering the target volatility as the target date approaches. For example, a target volatility scenario of "20%;1%" indicates that the target volatility is specified as 20% at the beginning of the 20-year hypothetical pension scheme and that it has an annual decreasing rate of 1%. To evaluate the performance of this target volatility strategy under different target volatilities, we examine 10 target volatility scenarios with starting target volatility ranges from 10% to 19%, covering the commonly used target volatility levels when applying the target volatility mechanism in pension investment practice. We also cap the maximum risky asset allocation to 150% in order to prevent the pension portfolio from being overexposed to risky assets. Admittedly, selecting a target volatility scenario and the maximum risky asset allocation level is an ambiguous process. Future studies might decide to choose different parameters for their target volatility scenarios and compare the results.

To achieve effective portfolio protection when encountering a turbulent event, the target volatility strategy adjusts the asset allocation of a pension portfolio between the risky and risk-free asset classes. The transaction cost, which would occur in the event of adjustment, could affect the ability of a pension portfolio to generate desirable returns. Since we adopt a daily rebalancing mechanism in our target volatility strategy, the influence of transaction cost on portfolio returns could be significant. Prior research points out a wide range of transaction costs in different trading scenarios. For example, (Hurst et al., 2017) suggest a transaction cost between 6bps to 34bps for trades on the equity asset class, based on an investment period between 1880 and 2013. (Clare et al., 2017) propose using 0.20% of the investment value (i.e. 20bps) as a one-way transaction cost when examining a trend-following methodology. Since the target volatility mechanism could also be considered as an asset allocation strategy that follows market volatility trends, we consider a 20bps transaction cost for the markets examined in this paper. A higher transaction cost would foreseeably lower the pension portfolio returns and yield a more conservative estimation. Hence, although beyond the scope of the present study, the performance of a target volatility strategy under different transaction cost levels merits future exploration

2.4 Data and Methodology

In this section, we introduce the investment regions, data, and different risk–return profiles that we use for our analyses.

2.4.1 Investment regions and data selection

In this study, we evaluate the performance of our newly proposed target volatility strategy in Japan, South Korea, and China. We use the World Population Aging 2019 report, obtained from the Department of Economic and Social Affairs of the United Nations, as a reference for selecting the markets for our analysis. According to the report, Japan, South Korea, and

China are projected to have the highest old-age dependency ratio by 2050 (United Nation, 2020). The old-age dependency ratio refers to the ratio between the number of people who are retired and the number of people who are working (Ediev et al., 2019). When a country has a high old-age dependency ratio, its working forces face a greater burden to support the living expenses of its elderly population (Pezzulo et al., 2017). Therefore, for the present study, we evaluate and compare the performance of a pension portfolio under the lifecycle strategy and the target volatility strategy in three countries that are predicted to have the highest old-age dependency ratio in the future. Our intention is not to use the markets of Japan, South Korea, and China to represent the Asian market as a whole but to only consider them as markets of interests in the present study. Future studies could select different markets for analyses.

The data for our analyses are extracted from Federal Reserve Economic Data and Bloomberg. We follow the existing literature to select daily historical times series of equities and bonds for the markets examined in this paper. We use the Nikkei 225 (Takahashi and Xu, 2016), Korea Composite Stock Price Index 200 (KOSPI; (Qin and Heo, 2017)), and Shanghai Stock Exchange Composite Index (SSE Composite; (Ahmed et al., 2017)) to represent the equity markets of Japan, South Korea, and China, respectively. For the bond markets, we use the commonly applied Bloomberg Barclays Aggregated family indices. Specifically, we use the Barclays Japanese Aggregate, the Barclays China Aggregate index (Kiymaz and Simsek, 2017; Lai, 2018; Martin and Sankaran, 2019), and the Korea Bond Index 120 (Lee et al., 2019) to represent the bond markets in the chosen investment regions. In this paper, we also examine the performances of the lifecycle and target volatility strategies from a global perspective, as a reference. Consequently, we use the MSCI World index (Umer et al., 2018) and the Barclays Global Aggregate index (Clare et al., 2019) to represent the world equity and bond markets, respectively.

2.4.2 Methodology

To evaluate the performance of the aforementioned two pension strategies, we consider a 20-year pension scheme—from March 2000 to March 2020. We select this time period because it includes the 2008 financial crisis as well as the most recent financial market turbulence of early 2020. We believe that these two recent financial market shocks offer suitable scenarios for examining whether or not our newly proposed target volatility strategy could provide pension-effective capital protection when facing market turbulence. Now, we calculate and compare different risk–return profiles of the pension portfolio under the lifecycle strategy and the target volatility strategy.

First, we calculate the risk–return ratio of the considered pension portfolio under each pension investment management strategy. The risk–return ratio is calculated as follows:

$$Risk-return\ ratio = \frac{R_t}{V_t^{(ER_t/|ER_t|)}} \tag{2.3}$$

The risk–return ratio could be viewed as a scaled portfolio return, which is a ratio between portfolio return and portfolio volatility. R_t represents the annual return of the pension portfolio in year t. V_t stands for the annualized volatility that is based on the daily returns in year t. ER_t represents the excessive return of the pension portfolio in year t, which is the difference between the annual return of the retirement portfolio and the annual risk-free rate. We apply an $ER_t/|ER_t|$ adjustment to the portfolio volatility so that the ratio will still be an effective measure when portfolio returns are negative (see Israelsen (2003); Israelsen et al. (2005) for details). The risk–return ratio shows the annual return that the pension portfolio could generate with every unit of risk. For the pension portfolio that is under lifecycle strategy and target volatility, we measure the risk–return ratio on a yearly basis over its 20-year retirement span and provide summary statistics for the risk–return ratio for the lifecycle and target volatility strategies in each of the studied investment regions.

Then, we calculate the 90%, 95%, and 99% value-at-risk (VAR) and conditional value-at-risk (CVAR) based on the one-year rolling annual returns of the pension portfolio under the two pension strategies. The 90%, 95%, and 99% VAR and CVAR measure the downside risk of the pension portfolio within a one-year investment span over the 20-year pension scheme. In other words, the 90%, 95%, and 99% VAR and CVAR demonstrate the worst 10%, 5%, and 1% expected annual returns of the pension portfolio within a one-year investment window over the 20-year pension span. In this paper, we use the historical method to calculate the VAR and CVAR at different levels. In other words, we first calculate the one-year rolling annual returns of the pension portfolio under each pension investment strategy (i.e. the lifecycle strategy and the target volatility strategy, respectively) and order the annual returns from lowest to highest. Subsequently, we use 90%, 95%, and 99% confidence levels to locate the corresponding VAR and CVAR values of the pension portfolio. When evaluating the performance of the pension portfolio based on the VAR and the CVAR, the CVAR is often considered to be a more conservative measure in comparison to the VAR. Moreover, since we use the historical approach to measure the VAR and the CVAR, the VAR is based on a single observation, while the CVAR is based on the average of several observations. Thus, we believe that the CVAR could be a more robust measure in comparison to the VAR.

We calculate annualized volatility using the daily returns of the pension portfolio in each financial year and generate the summary statistics for the annualized volatility of the pension portfolio under each pension strategy over the 20-year pension scheme. Annualized volatility captures the variation of portfolio returns in a financial year, which is often considered to be a proxy for measuring portfolio risk. We also create pension portfolio volatility paths to illustrate change in the annualized volatility of our pension portfolio under the lifecycle strategy and the target volatility strategy for each financial year over the 20-year pension scheme.

The annualized volatility of a portfolio shows the variation in portfolio returns over an investment period. However, annualized volatility does not differentiate upward variations from downward variations. In other words, annualized volatility may not be an informative measure if we wish to analyze the downside risk of our pension portfolio under the two pension strategies. Consequently, we also present the results for the downside deviation of the pension portfolio under each pension strategy. As an extended risk measure for annualized volatility, downside deviation measures the so-called "bad variations" in the returns that are below a specified threshold, which is often addressed as the minimum acceptable return. The downside deviation is calculated as follows:

$$DR_t = min(0, R_t - MAR_t) \tag{2.4}$$

$$DD = \sqrt{\frac{\sum (DR_t)^2}{n}} \tag{2.5}$$

In the above formulas, R_t represents the annual return and MAR_t represents the minimum acceptable return of the pension portfolio in year t. We set the minimum acceptable return as the annual risk-free rate (Elmesseary, 2014) in year t. The downside return is represented by DR_t, which equals the difference between the annual return of the pension portfolio and the minimum acceptable return if the difference is positive and is otherwise zero. Finally, n represents the total number of investment periods and DD stands for the downside variation of the pension portfolio over the 20-year pension span.

2.5 Results

Table 2.1 presents the summary statistics for the risk–return ratio of the pension portfolio under the two pension investment strategies in Japan, South Korea, and China.

Table 2.1: Summary statistics for the portfolio risk-return ratio

This table reports the summary statistics for the risk-return ratio of the pension portfolio under the lifecycle strategy and the target volatility strategy. The risk-return ratio is a ratio between the annualized return and the annualized volatility measured on a yearly basis over the twenty-year pension span. TVS stands for target volatility scenarios. A target volatility scenario of 10%; 0.50% indicates that the target volatility strategy starts at 6% at the beginning of the twenty-year pension scheme and then decreases by 0.50% every subsequent year.

	Lifecycle strategy	Target volatility strategy									
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
Max	3.725	11.063	10.132	9.350	8.683	8.109	7.609	7.170	6.782	6.435	6.125
Mean	0.896	1.317	1.257	1.211	1.172	1.138	1.111	1.090	1.068	1.046	1.025
Min	-0.058	-0.020	-0.025	-0.030	-0.035	-0.040	-0.045	-0.049	-0.053	-0.057	-0.060
Japan											
Max	2.656	3.748	3.451	3.204	2.995	2.829	2.835	2.861	2.883	2.907	2.935
Mean	0.911	1.012	0.977	0.948	0.924	0.904	0.886	0.871	0.859	0.847	0.838
Min	-0.081	-0.019	-0.023	-0.027	-0.031	-0.036	-0.040	-0.046	-0.051	-0.057	-0.063
South Korea											
Max	3.095	2.903	2.858	2.822	2.794	2.772	2.754	2.741	2.731	2.723	2.718
Mean	0.797	0.873	0.848	0.827	0.811	0.797	0.785	0.775	0.766	0.759	0.753
Min	-0.143	-0.011	-0.013	-0.016	-0.019	-0.023	-0.026	-0.030	-0.034	-0.039	-0.043
China											
Max	5.278	30.712	27.927	25.607	23.643	21.961	20.502	19.226	18.100	17.099	16.203
Mean	1.149	2.833	2.641	2.480	2.345	2.230	2.130	2.042	1.965	1.896	1.834
Min	-0.049	-0.015	-0.018	-0.021	-0.024	-0.028	-0.032	-0.037	-0.041	-0.045	-0.049

In terms of the risk–return ratio, the target volatility strategy (i.e. the lifecycle strategy with the target volatility overlay) outperforms the lifecycle strategy from a global perspective. In contrast to the lifecycle strategy, which has an average risk–return ratio of 0.896, the target volatility strategy shows an average risk–return ratio greater than 1.025 (i.e. when the target volatility scenario is 19%;0.95%) when looking at the global market.

In Japan and South Korea, we see that the pension portfolio that uses the target volatility strategy shows a higher risk–return ratio in comparison to that of the lifecycle strategy in some but not all target volatility scenarios. Specifically, in terms of the average risk–return ratio, the target volatility strategy outperforms the lifecycle strategy in Japan and South Korea when the starting target volatility level is smaller than 14%. When the starting target volatility level is smaller than 14%, the pension portfolio using the target volatility strategy results in an average risk–return ratio higher than 0.924 and 0.811 in comparison to the pension portfolio using the lifecycle strategy, which shows an average risk–return ratio of 0.911 and 0.797 in Japan and South Korea, respectively.

The target volatility strategy shows the most prominent results in China, where the target volatility strategy could generate a higher risk–return ratio relative to the lifecycle strategy in all the target volatility scenarios examined in this paper. Specifically, the average risk–return ratio of the pension portfolio is 1.149 under the lifecycle strategy while it is higher than 1.834 under the target volatility strategy, suggesting that the target volatility strategy could generate higher portfolio return relative to the lifecycle strategy when looking at the same portfolio risk level.

Another important finding relates to the minimum risk–return ratio. In the three examined investment regions, the minimum risk–return ratios of the pension portfolio under the target volatility strategy are equal to or higher than those of the portfolios under the lifecycle strategy. We further find that those minimum risk–return ratios occurred in 2008 when the financial markets in those investment regions were affected by the subprime financial crisis in the United States. This provides indirect evidence that, in contrast to the lifecycle strategy, the target volatility strategy could better protect pension portfolios when encountering financial market turbulence.

Overall, the summary statistics results suggest that, under certain target volatility scenarios, the pension portfolio using the target volatility strategy could generate a higher return than that using the lifecycle strategy in Japan, South Korea, and China. Attaching the target volatility mechanism to the lifecycle strategy could thus provide effective capital protection when facing unforeseeable market shocks.

In Table 2.2, we present results of the 99%, 95%, and 90% VAR for the one-year rolling annual returns of the pension portfolio over the 20-year pension scheme. The 99%, 95%, and 90% VAR measure the worst 1%, 5%, and 10% expected loss of the pension portfolio over a one-year investment window.

From a global perspective, there is a 1%, 5%, and 10% chance that the pension portfolio under the lifecycle strategy could expect a 27.260%, 20.714%, and 15.096% loss, respectively, in a one-year investment window over the 20-year pension scheme. When the starting target volatility level is smaller or equal to 12%, the pension portfolio using the target volatility strategy has a 1%, 5%, and 10% chance of losing less than 26.147%, 19.487%, and 13.740% in a one-year investment window. Therefore, with respect to the 99%, 95%, and 90% VAR, the target volatility strategy outperforms the lifecycle strategy when target volatility scenarios have a starting target volatility level that is equal to or less than 12%.

Table 2.2: The 90%, 95%, and 99% portfolio value-at-risk results

This table reports the 90%, 95%, and 99% VAR of the pension portfolio under the lifecycle strategy and the target volatility strategy. The 90%, 95%, and 99% VAR are calculated based on the one-year annualized rolling returns of the pension portfolio over the twenty-year pension span, which measures the expected loss of the pension portfolio over a one-year investment window. TVS stands for target volatility scenarios. A target volatility scenario of 10%; 0.50% indicates that the target volatility strategy starts at 6% at the beginning of the twenty-year pension scheme and then decreases by 0.50% every subsequent year.

	Lifecycle strategy	Target volatility strategy									
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
99% VAR	-27.260%	-21.486%	-23.880%	-26.147%	-28.274%	-30.309%	-31.982%	-33.091%	-33.924%	-34.953%	-35.540%
95% VAR	-20.714%	-15.916%	-17.754%	-19.487%	-21.225%	-22.832%	-24.251%	-25.314%	-26.123%	-26.865%	-27.439%
90% VAR	-15.096%	-11.324%	-12.526%	-13.740%	-14.911%	-16.034%	-17.166%	-18.018%	-18.787%	-19.486%	-19.933%
Japan											
99% VAR	-29.585%	-18.597%	-20.287%	-21.941%	-23.562%	-25.149%	-26.703%	-28.225%	-29.716%	-31.178%	-32.610%
95% VAR	-22.078%	-11.655%	-12.762%	-13.847%	-14.917%	-15.975%	-17.020%	-18.051%	-19.073%	-20.095%	-21.102%
90% VAR	-14.533%	-6.602%	-7.247%	-7.907%	-8.545%	-9.204%	-9.850%	-10.494%	-11.151%	-11.780%	-12.420%
South Korea											
99% VAR	-26.583%	-7.805%	-8.941%	-10.056%	-11.161%	-12.306%	-13.383%	-14.452%	-15.547%	-16.589%	-17.617%
95% VAR	-15.133%	-2.789%	-3.534%	-4.246%	-4.947%	-5.655%	-6.323%	-7.004%	-7.707%	-8.395%	-9.054%
90% VAR	-4.483%	0.013%	-0.223%	-0.536%	-0.939%	-1.339%	-1.706%	-2.114%	-2.543%	-2.934%	-3.317%
China											
99% VAR	-39.028%	-16.965%	-18.718%	-20.435%	-22.100%	-23.757%	-25.316%	-26.850%	-28.300%	-29.528%	-30.687%
95% VAR	-20.848%	-12.542%	-13.952%	-15.241%	-16.557%	-17.889%	-19.298%	-20.495%	-21.663%	-22.695%	-23.561%
90% VAR	-14.690%	-9.537%	-10.687%	-11.827%	-12.962%	-14.076%	-15.146%	-16.123%	-17.086%	-17.907%	-18.755%

In Japan, South Korea, and China, using the target volatility strategy shows higher 99%, 95%, and 90% VAR values in certain target volatility scenarios in comparison to adopting the lifecycle strategy. The pension portfolio using the lifecycle strategy would have a 1%, 5%, and 10% likelihood of losing 29.585%, 22.078%, and 14.533%, respectively, over a one-year investment span in Japan. Given those confidence levels, the target volatility strategy could lower the expected losses of a pension portfolio when the starting target volatility level is smaller or equal to 16%, resulting in a 1%, 5%, and 10% chance of losing less than 28.225%, 18.051%, and 10.494% respectively. In South Korea, the target volatility strategy outperforms the lifecycle strategy in all target volatility scenarios. There is a 1%, 5%, and 10% chance that the pension portfolio would have less than 17.617%, 9.054%, and 3.317% loss when using the target volatility strategy in comparison to a loss of 26.583%, 15.133%, and 4.483%, respectively, when choosing the lifecycle strategy in South Korea. In China, the pension portfolio using the target volatility strategy shows a 1%, 5%, and 10% likelihood of one-year expected loss (i.e. 23.757%, 17.889%, and 14.076%, respectively) in comparison to that of the lifecycle strategy (i.e. 39.028%, 20.848%, and 14.690%, respectively) when the target volatility scenario has a starting target volatility level that is equal to or less than 14%.

From the VAR results, we observe that the VAR decreases as the starting target volatility level increases, indicating that there could be a negative association between the target volatility level and the one-year expected loss of a pension portfolio. This is not surprising because a higher target volatility level allows for a greater portion of risky assets in the pension portfolio. Hence, selecting a suitable target volatility level is an essential step when employing the target volatility strategy.

As an extended VAR risk measure, the CVAR represents the average expected loss of a pension portfolio given a confidence level. For instance, the 95% CVAR of the annual return is the average of the expected annual loss of a portfolio in the worst 5% annual return cases. Table 2.3 presents the results of 99%, 95%, and 90% CVAR for the one-year rolling annual returns of our pension portfolio under the two pension strategies.

The CVAR results are similar to the VAR results. In general, under certain target volatility scenarios, the pension portfolio using the target volatility strategy shows a lower one-year expected loss in comparison to the lifecycle strategy. Consistent with the VAR results, the target volatility strategy outperforms the lifecycle strategy when the starting target volatility level is equal to or less than 12% when looking at the global market. In Japan, the pension portfolio under the lifecycle strategy has a 1%, 5%, and 10% chance of encountering an average one-year loss of 32.443%, 26.255%, and 22.285% in comparison to less than 31.943%, 26.594%, and 20.603% when using the target volatility strategy that has a starting target volatility level of less than 18%. With respect to the CVAR, the pension portfolio using the target volatility strategy would expect a 1%, 5%, and 10% chance of loss 20.284%, 14.405%, and 10.058% or less on average relative to 29.332%, 21.922%, and 15.973% when using the lifecycle strategy in South Korea. In China, the target volatility strategy also outperforms the lifecycle strategy in all target volatility scenarios. Under the target volatility strategy, the one-year average expected loss is less than 33.252%, 27.697%, and 24.361% based on the 99%, 95%, and 90% CVAR, respectively, while the one-year expected loss of the pension portfolio under the lifecycle strategy is 41.190%, 31.194%, and 24.367%, respectively.

Table 2.3: The 90%, 95%, and 99% portfolio conditional value-at-risk results

This table reports the 90%, 95%, and 99% VAR of the pension portfolio under the lifecycle strategy and the target volatility strategy. The 90%, 95%, and 99% VAR are calculated based on the one-year annualized rolling returns of the pension portfolio over the twenty-year pension span, which measures the expected loss of the pension portfolio over a one-year investment window. TVS stands for target volatility scenarios. A target volatility scenario of 10%; 0.50% indicates that the target volatility strategy starts at 6% at the beginning of the twenty-year pension scheme and then decreases by 0.50% every subsequent year.

	Lifecycle strategy	Target volatility strategy									
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
99% CVAR	-28.858%	-23.068%	-25.458%	-27.773%	-29.964%	-32.069%	-33.859%	-35.187%	-36.286%	-37.313%	-38.107%
95% CVAR	-24.877%	-19.256%	-21.326%	-23.345%	-25.276%	-27.146%	-28.772%	-30.036%	-31.068%	-32.026%	-32.726%
90% CVAR	-21.211%	-16.409%	-18.178%	-19.907%	-21.573%	-23.168%	-24.596%	-25.721%	-26.652%	-27.521%	-28.156%
Japan											
99% CVAR	-32.443%	-19.152%	-20.877%	-22.565%	-24.215%	-25.830%	-27.409%	-28.954%	-30.465%	-31.943%	-33.389%
95% CVAR	-26.255%	-15.692%	-17.142%	-18.565%	-19.964%	-21.338%	-22.687%	-24.013%	-25.315%	-26.594%	-27.850%
90% CVAR	-22.285%	-12.008%	-13.138%	-14.252%	-15.349%	-16.431%	-17.497%	-18.548%	-19.583%	-20.603%	-21.609%
South Korea											
99% CVAR	-29.332%	-9.298%	-10.591%	-11.865%	-13.121%	-14.358%	-15.579%	-16.781%	-17.965%	-19.133%	-20.284%
95% CVAR	-21.922%	-5.823%	-6.819%	-7.805%	-8.779%	-9.743%	-10.696%	-11.639%	-12.571%	-13.493%	-14.405%
90% CVAR	-15.973%	-3.439%	-4.195%	-4.949%	-5.699%	-6.442%	-7.178%	-7.908%	-8.631%	-9.348%	-10.058%
China											
99% CVAR	-41.190%	-18.571%	-20.445%	-22.255%	-24.016%	-25.780%	-27.465%	-29.109%	-30.633%	-32.020%	-33.252%
95% CVAR	-31.194%	-15.027%	-16.624%	-18.157%	-19.645%	-21.175%	-22.668%	-24.099%	-25.415%	-26.602%	-27.697%
90% CVAR	-24.367%	-12.975%	-14.409%	-15.796%	-17.146%	-18.527%	-19.880%	-21.164%	-22.341%	-23.398%	-24.361%

Next, we present the descriptive statistics for the annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy. We also show the pension portfolio volatility paths for the annualized volatility of the pension portfolio under each strategy using the 15%;0.75% target volatility scenario as an example. The pension portfolio volatility paths visualize the differences and changes in the annualized volatility under the two pension investment strategies by year over the 20-year pension scheme.

Based on the results shown in Table 2.4, the pension portfolio using the target volatility strategy shows a significantly lower maximum, average, and minimum annualized volatility in comparison to that of the lifecycle strategy over the 20-year pension scheme. In contrast to the lifecycle strategy, which shows an average annualized volatility of 8.833%, the target volatility strategy results in an average annualized volatility that is smaller than 6.856% when looking at the world market. The difference in the average annualized volatility between the lifecycle strategy and the target volatility strategy is more pronounced in the three Asian markets. The pension portfolio using the lifecycle strategy has an average annualized volatility of 11.930%, 12.920%, and 11.724% in Japan, South Korea, and China, respectively, while the pension portfolio using the target volatility strategy could maintain the average annualized volatility at a level smaller than 6.635%, 6.646%, and 6.655%, respectively.

Figures 2.1 to 2.4 show the pension portfolio volatility paths of the pension portfolio under the lifecycle strategy and the target volatility strategy over the 20-year pension scheme. Here, we use the 15%;0.75% target volatility scenario as an example because it represents a middle-level target volatility case among the target volatility scenarios we examine in this paper. The annualized volatility changing paths for other target volatility scenarios are similar to the example we demonstrate here.

Table 2.4: Summary statistics for the portfolio annualized volatility

This table reports the summary statistics for the annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy. The annualized volatility of the pension portfolio is measured on a yearly basis over the twenty-year pension span. TVS stands for target volatility scenarios. A target volatility scenario of 10%; 0.50% indicates that the target volatility strategy starts at 6% at the beginning of the twenty-year pension scheme and then decreases by 0.50% every subsequent year.

	Lifecycle strategy	Target volatility strategy									
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
Max	21.110%	10.198%	11.170%	12.145%	13.099%	14.039%	14.923%	15.722%	16.475%	17.190%	17.832%
Mean	8.833%	3.836%	4.216%	4.593%	4.961%	5.319%	5.664%	5.988%	6.294%	6.583%	6.856%
Min	2.932%	0.176%	0.193%	0.210%	0.227%	0.244%	0.261%	0.278%	0.295%	0.312%	0.330%
Japan											
Max	26.826%	9.970%	10.967%	11.964%	12.961%	13.958%	14.955%	15.953%	16.950%	17.947%	18.944%
Mean	11.930%	3.501%	3.851%	4.202%	4.552%	4.902%	5.251%	5.599%	5.946%	6.291%	6.635%
Min	2.053%	0.092%	0.101%	0.110%	0.120%	0.129%	0.138%	0.147%	0.156%	0.166%	0.175%
South Korea											
Max	44.044%	9.931%	10.924%	11.917%	12.910%	13.903%	14.896%	15.888%	16.881%	17.874%	18.867%
Mean	12.920%	3.498%	3.848%	4.197%	4.547%	4.897%	5.247%	5.597%	5.947%	6.296%	6.646%
Min	0.561%	0.027%	0.030%	0.032%	0.034%	0.036%	0.039%	0.041%	0.043%	0.046%	0.048%
China											
Max	25.061%	9.835%	10.756%	11.693%	12.646%	13.573%	14.491%	15.400%	16.258%	17.098%	17.902%
Mean	11.724%	3.567%	3.921%	4.274%	4.626%	4.974%	5.319%	5.663%	6.001%	6.332%	6.655%
Min	1.220%	0.032%	0.035%	0.038%	0.042%	0.045%	0.048%	0.051%	0.054%	0.058%	0.061%

For all the markets examined in the present study, we see that both strategies (i.e. the lifecycle strategy and the target volatility strategy) are able to gradually lower the annualized volatility of the pension portfolio as it approaches the end of its 20-year pension span. This pattern ties well with the pension management concept held by the traditional lifecycle strateg—securing the pension portfolio when approaching the target date (e.g. the retirement date). Nonetheless, for the year 2008, the annualized volatility of the pension portfolio increased dramatically under the lifecycle strategy, while it remained at a constant level for the portfolio using the target volatility strategy in all four investment regions. This finding is rather important because it shows that the target volatility strategy can provide effective capital protection for a pension portfolio when encountering shocking financial events, such as the 2008 financial crisis. This is further confirmed by the volatility changing paths for China, which experienced another market turbulence in 2015. Once again, the annualized volatility of the pension portfolio using the lifecycle strategy jumped up, while that using the target volatility strategy remained unchanged. We also see the same pattern in the most recent global market, where the annualized volatility of the pension portfolio using the lifecycle strategy increases in 2020 due to the market shock caused by the COVID-19 pandemic. Meanwhile, the annualized volatility of the pension portfolio using the target volatility strategy is controlled and at a low level. Therefore, it is evident that the target volatility strategy could provide more effective portfolio protection than the lifecycle strategy in the investment regions examined in this paper.

Figure 2.1: Volatility changing path: World

This diagram shows change in annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy by year over the twenty-year pension span. The dashed line represents the annualized volatility of the pension portfolio under the lifecycle strategy and the solid line represents the annualized volatility of the pension portfolio under the target volatility strategy.

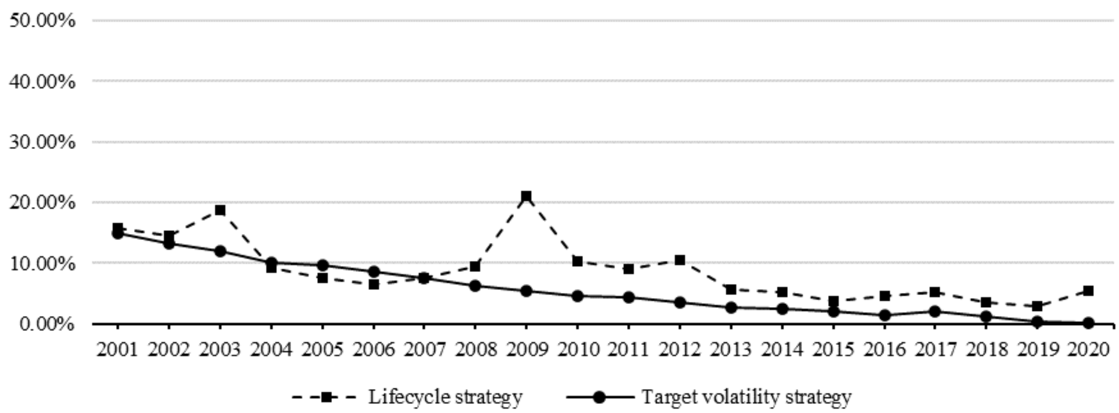


Figure 2.2: Volatility changing path: Japan

This diagram shows change in annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy by year over the twenty-year pension span. The dashed line represents the annualized volatility of the pension portfolio under the lifecycle strategy and the solid line represents the annualized volatility of the pension portfolio under the target volatility strategy.

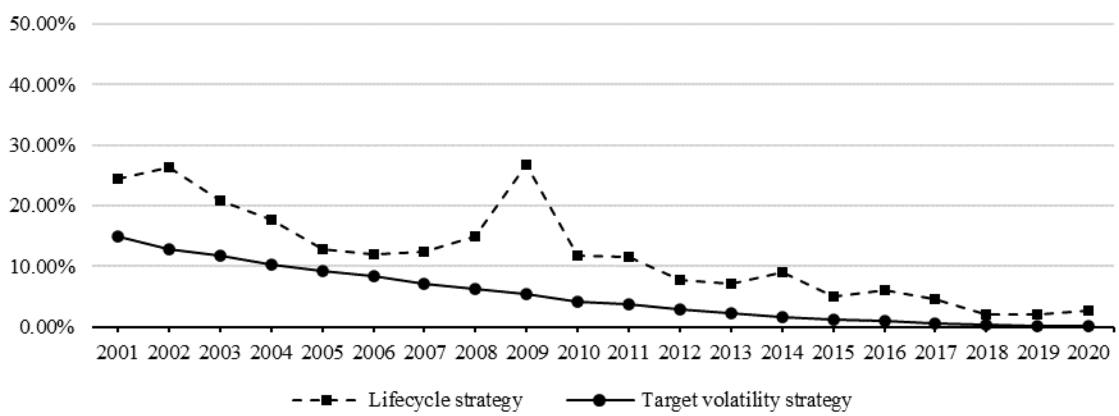


Figure 2.3: Volatility changing path: South Korea

This diagram shows change in annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy by year over the twenty-year pension span. The dashed line represents the annualized volatility of the pension portfolio under the lifecycle strategy and the solid line represents the annualized volatility of the pension portfolio under the target volatility strategy.

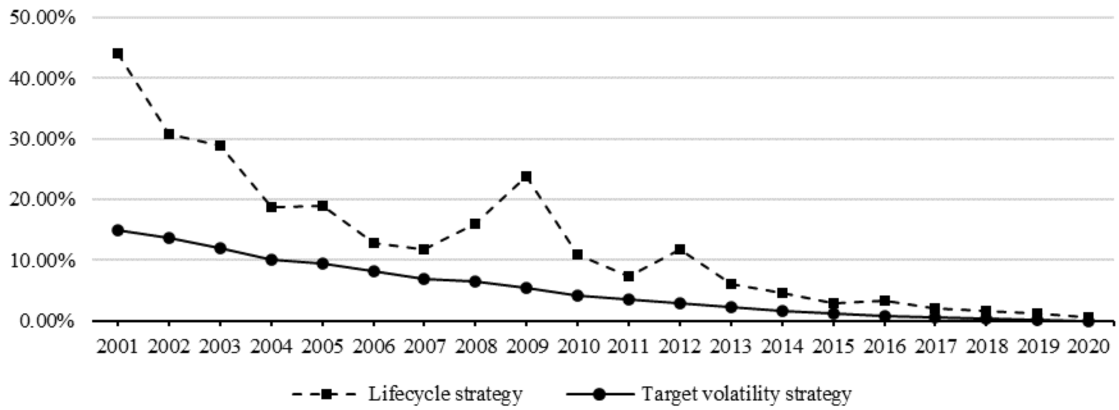
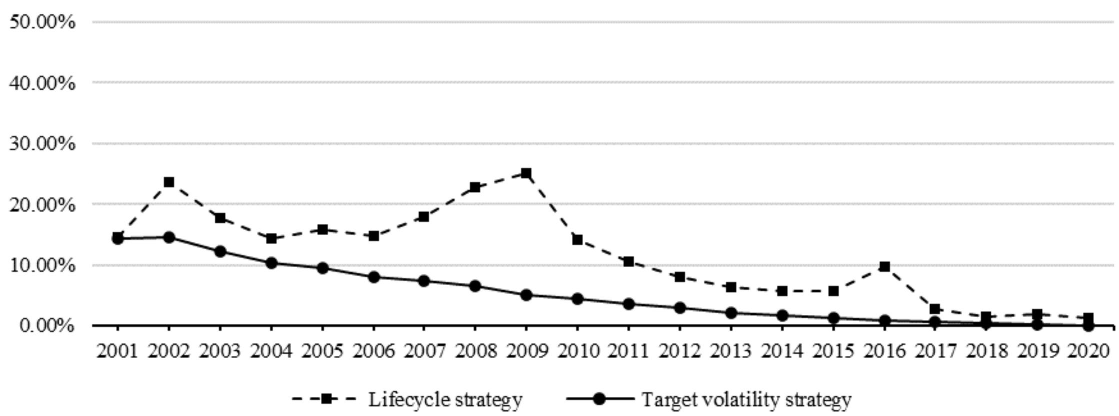


Figure 2.4: Volatility changing path: China

This diagram shows change in annualized volatility of the pension portfolio under the lifecycle strategy and the target volatility strategy by year over the twenty-year pension span. The dashed line represents the annualized volatility of the pension portfolio under the lifecycle strategy and the solid line represents the annualized volatility of the pension portfolio under the target volatility strategy.



Finally, we present the downside deviation of the pension portfolio over the twenty-year pension scheme in Table 5.

Table 2.5: Portfolio downside deviation results

This table reports the downside deviation of the pension portfolio under the lifecycle strategy and the target volatility strategy. Results are calculated based on portfolio volatilities associated with negative excess portfolio returns over the twenty-year pension span. A target volatility scenario of 10%; 0.50% indicates that the target volatility strategy starts at 6% at the beginning of the twenty-year pension scheme and then decreases by 0.50% every subsequent year.

	Lifecycle strategy	Target volatility strategy									
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World	10.427%	7.766%	8.469%	9.156%	9.811%	10.447%	11.002%	11.457%	11.827%	12.186%	12.485%
Japan	10.491%	5.483%	5.984%	6.476%	6.960%	7.437%	7.907%	8.368%	8.823%	9.271%	9.711%
South Korea	12.161%	4.846%	5.293%	5.733%	6.167%	6.595%	7.017%	7.432%	7.842%	8.245%	8.643%
China	9.261%	6.334%	6.903%	7.449%	7.979%	8.524%	9.060%	9.573%	10.050%	10.483%	10.883%

Over the 20-year pension scheme, we see that the target volatility strategy could better control the portfolio downside deviation under all target volatility scenarios in both Japan and South Korea. The downside deviation of the pension portfolio under the lifecycle strategy is 10.491% and 12.161%, while its downside deviation under the target volatility strategy is smaller than 9.711% and 8.643% in Japan and South Korea, respectively. The target volatility strategy outperforms the lifecycle strategy when the target volatility scenario has a starting target volatility level that is equal to or less than 13% when looking at the global market. In China, using the lifecycle strategy results in a downside deviation of 9.261% and in a downside deviation of less than 9.060% when using the target volatility strategy with a starting volatility level that is equal to or less than 15%.

2.6 Robustness Check

To further examine whether or not attaching the target volatility mechanism to the lifecycle strategy could better control portfolio risk, we first follow (Elton et al., 2016) and use Jensen's alpha regression approach (Jensen, 1968) to evaluate the beta of the pension portfolio under both the lifecycle strategy and the target volatility strategy. According to our results on the annualized volatility of the pension portfolio under these two strategies, we find that the pension portfolio using the target volatility strategy results in significantly lower annualized volatility than when using the lifecycle strategy over the 20-year pension scheme. Hence, in this regression model, we focus on the beta of the pension portfolio because the portfolio alpha is not a suitable measure in Jensen's regression for a comparison between portfolios with different levels of risk (Cogneau and Hübner, 2009). Specifically, we regress the excess return of the pension portfolio under each pension strategy on the excess return of the local equity market at time t as follows:

$$R_t^P - r_t = \alpha_P + \beta_P \cdot (R_t^M - r_t) + \varepsilon_t^P \tag{2.6}$$

In this empirical model, R_t^P represents the one-year rolling annualized return of the pension portfolio, while r_t stands for the annual risk-free rate at time t. R_t^M represents the one-year rolling annualized return of the equity market, α_P is Jensen's alpha, and β_P stands for the portfolio's beta.

Table 2.6 presents the regression results. Column 1 shows the regression results for the pension portfolio under the lifecycle strategy and columns 2 to 11 report the regression results for the pension portfolio under the target volatility strategy using different target volatility scenarios.

We see that the beta coefficients of the excess return of equity markets (i.e. β_P for $R_t^M - rt$) are smaller than one and are statistically significant at the 0.01 significance level, suggesting that both the lifecycle strategy and the target volatility strategy could lower portfolio risk by including risk-free assets in the pension portfolio in the four investment regions. Specifically, when using the lifecycle strategy, the portfolio beta is 0.676, 0.528, 0.683, and 0.525 in the world, Japan, South Korea, and China, respectively. When combining the lifecycle strategy with the target volatility mechanism (i.e. the target volatility strategy), we see that the beta coefficients are further reduced to a significantly lower level. Using the target volatility strategy, the maximum portfolio beta is 0.666 in the world, 0.388 in Japan, 0.450 in South Korea, and 0.300 in China, all of which are lower than those obtained using the lifecycle strategy. This result confirms that adding the target volatility mechanism to the lifecycle strategy could further mitigate market risk and provide additional capital protection to a pension portfolio.

Table 2.6: Jensen's alpha regression results

The Jensen's alpha regression model regresses the excess returns of a pension portfolio on the excess returns of equity markets in each investment region. In this table, column (1) reports the regression results for the lifecycle strategy in the four investment regions. Column (2) to (11) report the regression results for the target volatility strategy under each of the target volatility scenarios (i.e., from 10%;0.50% to 19%;0.95%) in the four investment regions. Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

	(1) Lifecycle strategy	(2) Target volatility strategy	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
β_P	0.676	0.393	0.431	0.469	0.504	0.538	0.570	0.599	0.624	0.646	0.666
	(0.004)***	(0.004)***	(0.005)***	(0.005)***	(0.006)***	(0.006)***	(0.006)***	(0.007)***	(0.007)***	(0.007)***	(0.007)***
α_P	-0.008	-0.019	-0.021	-0.022	-0.023	-0.024	-0.025	-0.026	-0.026	-0.027	-0.028
	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.877	0.617	0.617	0.617	0.618	0.620	0.622	0.625	0.631	0.638	0.647
Japan											
β_P	0.528	0.206	0.226	0.246	0.267	0.287	0.307	0.328	0.348	0.368	0.388
	(0.004)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***	(0.004)***	(0.004)***	(0.004)***	(0.005)***	(0.005)***
α_P	-0.002	-0.013	-0.014	-0.015	-0.016	-0.017	-0.018	-0.019	-0.020	-0.020	-0.021
	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.795	0.549	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550	0.550

Table 2.6: Jensen's alpha regression results (Continued)

The Jensen's alpha regression model regresses the excess returns of a pension portfolio on the excess returns of equity markets in each investment region. In this table, column (1) reports the regression results for the lifecycle strategy in the four investment regions. Column (2) to (11) report the regression results for the target volatility strategy under each of the target volatility scenarios (i.e., from 10%;0.50% to 19%;0.95%) in the four investment regions. Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

	(1) Lifecycle strategy	(2) Target volatility strategy	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
South Korea											
β_P	0.683	0.231	0.255	0.279	0.303	0.327	0.351	0.376	0.400	0.425	0.450
	(0.003)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***
α_P	-0.002	-0.003	-0.003	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
	(0.001)***	(0.000)***	(0.000)***	(0.000)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.889	0.770	0.770	0.769	0.769	0.769	0.769	0.769	0.769	0.768	0.768
China											
β_P	0.525	0.151	0.167	0.183	0.200	0.216	0.233	0.250	0.266	0.283	0.300
	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***
α_P	-0.008	-0.021	-0.023	-0.025	-0.026	-0.028	-0.030	-0.031	-0.033	-0.034	-0.035
	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***	(0.001)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.928	0.625	0.631	0.637	0.642	0.647	0.653	0.660	0.668	0.676	0.686

Next, we use the Henriksson and Merton (Henriksson and Merton, 1981) measure to examine whether or not a pension portfolio could benefit from attaching the target volatility mechanism to the lifecycle strategy and result in better market timing skill. In other words, we estimate whether the target volatility strategy (i.e. attaching the target volatility mechanism to the lifecycle strategy) could better adjust portfolio asset classes based on market performance in comparison to the lifecycle strategy. The H&M regression model is shown as follows:

$$R_t^P - r_t = \alpha_P + \beta_P \cdot (R_t^M - r_t) + \gamma_t \cdot D_t \cdot (R_t^M - r_t) + \varepsilon_t^P \quad (2.7)$$

In the H&M regression model, R_t^P, r_t, α_P, and R_t^M represent the one-year rolling annualized return of the pension portfolio, the annual risk-free rate, the portfolio's alpha, and the one-year rolling annualized return of the equity market at time t, respectively. D_t is a dummy variable that is equal to one if the difference $R_t^M - r_t$ is positive and zero otherwise. A positive and statistically significant coefficient, γ_t, suggests that the pension portfolio could benefit from good market timing skills being embedded in an investment strategy, resulting in higher portfolio return.

Table 2.7 shows the H&M regression results for the pension portfolio under the lifecycle strategy and the target volatility strategy.

Looking at the global market, we see that the γ_t coefficient of the pension portfolio using the lifecycle strategy is negative and statistically significant. Using the target volatility strategy, the γ_t coefficient turns positive under all target volatility scenarios, suggesting that attaching the target volatility mechanism to the lifecycle strategy improves its market timing skill. Both the lifecycle strategy and the target volatility strategy show positive and statistically significant γ_t in Japan. Nonetheless, the γ_t of the pension portfolio under the lifecycle strategy is 0.009, while it is greater than 0.012 when under the target volatility strategy (i.e. when the target volatility scenario is 10%;0.50%), which suggests that the target volatility strategy outperforms the lifecycle strategy with respect to the market timing skill. In South Korea, the pension portfolio under the two pension investment strategies results in a negative γ_t. However, the γ_t of the pension portfolio when using the target volatility strategy (i.e. greater than -0.021) is higher than when using the lifecycle strategy (i.e. -0.051). Finally, in China, the H&M regression shows similar results to the global market, where the γ_t is negative and statistically significant under the lifecycle strategy but turns positive when the target volatility mechanism is attached to the lifecycle strategy, showing that we could expect a higher portfolio return from embedding a better market timing skill in the target volatility strategy.

Overall, our model-based analyses suggest that, in contrast to the lifecycle strategy, the target volatility strategy, which contains an additional target volatility mechanism, could additionally control portfolio risk and adjust portfolio asset allocation better on the basis of market performance. These results are consistent with our main analysis findings and, consequently, we conclude that our results are robust.

Table 2.7: Henriksson & Merton (H&M) regression results

The H&M model regresses the excess returns of a pension portfolio on the excess returns of an equity market and a dummy variable, which equals one if the excess return of the equity market is positive and zero otherwise. In this table, column (1) reports the regression results for the lifecycle strategy in the four investment regions. Column (2) to (11) report the regression results for the target volatility strategy under each of the target volatility scenarios (i.e., from 10%;0.50% to 19%;0.95%) in the four investment regions. In the regression results, we focus on the coefficient γ_t. The coefficient α_P and the constant term in the regression model are omitted from this regression output. Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

	(1) Lifecycle strategy	(2) Target volatility strategy	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
World											
β_P	0.756	0.378	0.415	0.449	0.481	0.510	0.538	0.564	0.586	0.608	0.630
	(0.006)***	(0.007)***	(0.008)***	(0.009)***	(0.009)***	(0.010)***	(0.011)***	(0.011)***	(0.011)***	(0.012)***	(0.012)***
γ_t	-0.036	0.007	0.007	0.009	0.010	0.012	0.015	0.016	0.017	0.017	0.016
	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***	(0.004)***	(0.004)***	(0.004)***	(0.004)***	(0.004)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.884	0.617	0.617	0.617	0.619	0.621	0.623	0.626	0.632	0.639	0.648
Japan											
β_P	0.513	0.186	0.205	0.223	0.242	0.261	0.280	0.299	0.318	0.336	0.355
	(0.006)***	(0.004)***	(0.005)***	(0.005)***	(0.005)***	(0.006)***	(0.006)***	(0.007)***	(0.007)***	(0.007)***	(0.008)***
γ_t	0.009	0.012	0.013	0.014	0.014	0.015	0.016	0.017	0.018	0.019	0.019
	(0.003)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.795	0.553	0.553	0.553	0.553	0.552	0.553	0.553	0.552	0.553	0.553

Table 2.7: Henriksson & Merton (H&M) regression results (Continued)

The H&M model regresses the excess returns of a pension portfolio on the excess returns of an equity market and a dummy variable, which equals one if the excess return of the equity market is positive and zero otherwise. In this table, column (1) reports the regression results for the lifecycle strategy in the four investment regions. Column (2) to (11) report the regression results for the target volatility strategy under each of the target volatility scenarios (i.e., from 10%;0.50% to 19%;0.95%) in the four investment regions. In the regression results, we focus on the coefficient γ_t. The coefficient α_P and the constant term in the regression model are omitted from this regression output. Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

	(1) Lifecycle strategy	(2) Target volatility strategy	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TVS		10%;0.50%	11%;0.55%	12%;0.60%	13%;0.65%	14%;0.70%	15%;0.75%	16%;0.80%	17%;0.85%	18%;0.90%	19%;0.95%
South Korea											
β_P	0.770	0.247	0.273	0.299	0.325	0.351	0.377	0.404	0.431	0.458	0.485
	(0.005)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***	(0.004)***	(0.004)***	(0.004)***	(0.005)***	(0.005)***	(0.005)***
γ_t	-0.051	-0.009	-0.010	-0.011	-0.013	-0.014	-0.015	-0.017	-0.018	-0.019	-0.021
	(0.002)***	(0.001)***	(0.001)***	(0.001)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***	(0.002)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.900	0.772	0.772	0.772	0.772	0.772	0.772	0.772	0.772	0.772	0.772
China											
β_P	0.534	0.131	0.146	0.161	0.176	0.191	0.206	0.222	0.239	0.255	0.272
	(0.003)***	(0.002)***	(0.002)***	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.004)***	(0.004)***
γ_t	0.013	0.030	0.032	0.034	0.036	0.038	0.040	0.041	0.042	0.043	0.043
	(0.003)***	(0.002)***	(0.002)***	(0.002)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***	(0.003)***
n	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957	4957
R^2	0.929	0.642	0.647	0.652	0.657	0.661	0.666	0.672	0.679	0.686	0.695

2.7 Discussion

Pension scholars and practitioners are continuously seeking reliable long-term investment strategies that can help them keep their promises. In the present study, we examine the possibility of improving the traditional lifecycle pension investment concept by attaching a target volatility mechanism to it. Based on our numerical results, we see that a combination of the lifecycle pension investment strategy and the target volatility strategy outperforms the lifecycle strategy on its own in terms of portfolio risk control. This could largely be attributed to the flexible asset management concept of the target volatility strategy. A pension portfolio that operates under a traditional lifecycle strategy often begins with a large portion of risky assets, gradually reducing the amount of risky assets in the pension portfolio over the course of the pension investment period (Tang and Lin, 2015). This fixed glide-path mechanism secures the pension portfolio as it moves toward the target date but it also exposes the portfolio to market risk during the beginning stage of the investment (Pfau, 2010). In other words, we can see the lifecycle strategy as being a time-dependent long-term investment strategy with a target date constraint. In contrast to the lifecycle strategy, the target volatility strategy is a performance-driven investment strategy that adjusts the asset allocation of a pension portfolio on the basis of market volatilities and target volatilities. Thus, although pension portfolios under the target volatility strategy also have a target date, the embedded asset allocation mechanism is time-independent. This allows the target volatility strategy to adjust the asset allocation anytime during the pension accumulation stage, to maintain the portfolio risk at a constant level (Cirelli et al., 2017), and to mitigate the portfolio downside risk (Benson et al., 2014).

In terms of pension portfolio returns, we see that the combined strategy (i.e. the lifecycle strategy that uses a target volatility overlay) could generate higher returns-in some but not all target volatility scenarios—over the 20-year pension scheme in comparison to the lifecycle strategy on its own. As we show in our results, the target volatility strategy outperforms the lifecycle strategy in China for all target volatility scenarios with respect to the risk-adjusted returns but the same strategy shows better performance in Japan and South Korea only under some but not all target volatility scenarios. This is not surprising because each market is different and it is possible that the target volatility mechanism is more effective in certain market contexts than in others. Hence, a detailed analysis that looks for investment regions in which these investment strategies could be implemented is necessary. Nonetheless, the benefits of the target volatility strategy also stand out in the following aspects. First, unlike the pension portfolio under the lifecycle strategy, which is dominated by risk-free assets when approaching the target date, the pension portfolio under the target volatility strategy is able to adjust its asset allocation at different time points during the entire pension accumulation stage, thus allowing for the possibility of investing in risky assets even when approaching the target date. Second, the target volatility strategy allows for a certain amount of leverage so that the pension portfolio could benefit from a high-return equity market when the market conditions are desirable (i.e. when the realized market volatility is lower than the target volatility). This leverage mechanism is also lacking in the traditional lifecycle strategy, which allows for maximum 100% risky asset allocation in the pension portfolio.

Despite the findings presented in this paper, it is also worth mentioning that the study has several limitations. First, we attempt to examine whether attaching the target volatility mechanism to the traditional lifecycle pension concept can yield better pension investment outcomes. Consequently, we select a simple linear lifecycle strategy even though there are other more sophisticated lifecycle pension strategies and investment vehicles available in the markets. Second, in order to show the effective asset protection embedded in the target volatility strategy, we intentionally set up a pension scheme with a 20-year span that included the 2008 financial crisis and the recent market turbulence of early 2020. As a result, our analyses derive from historical time-series data. Finally, we consider a pension portfolio that has a simple asset composition (i.e. equities and bonds), although there are more complex financial derivatives available for a pension portfolio.

Given the findings in and limitations of this study, future research could examine the suitability of combining other dynamic risk management components, such as the mean-variance asset allocation strategy (Dang et al., 2017) and the liability-based portfolio management strategy (Crook, 2019) with the classical lifecycle concept. Furthermore, instead of using historical time-series data to conduct corresponding analyses, a future study could also investigate forward-looking scenarios. Finally, asset allocation is just one of the factors that can affect the performance of a pension portfolio. In other words, pension investment strategies also need to confront other factors, such as longevity risk (Simsek et al., 2018) and inflation (Tang et al., 2018), which could shape the sustainability of a pension portfolio.

2.8 Conclusion

In this paper, we propose a new pension management investment strategy that combines a simple lifecycle pension concept with a target volatility mechanism in three investment regions (i.e. Japan, South Korea, and China) that have the highest predicted old-age dependency ratio. We find that, in contrast to the sole use of the simple lifecycle strategy, the combined strategy that uses the target volatility mechanism could control portfolio downside risks better and generate higher portfolio returns. We also show that the combined strategy that uses the target volatility overlay could provide more effective pension capital protection in comparison to the sole use of the traditional lifecycle pension strategy when facing a sudden market drawdown. Our results are robust and take transaction costs into consideration. Therefore, this study shows that attaching a target volatility mechanism to the lifecycle strategy could be an option in pension markets.

Enhancing Retirement Coverage Using The Target Volatility Investment Strategy

3.1 Introduction

As population aging becomes a global phenomenon, the problem of optimizing the retirement planning has attracted a lot of attention in financial industry and academic discussion. According to the World Population Aging 2019 report from the United Nations, approximately 703 million people were aged 65 or older, and this number is projected to reach 1.5 billion by 2050 (United Nation, 2020). Recently, new challenges have raised in the retirement management domain due to the unprecedented market crisis triggered by the COVID-19 pandemic (Baker et al., 2020), and a persistent low interest rate environment in marketplace (Rubio and Yao, 2020). This may lead to a myriad of issues to different retirement investment vehicles since most of the retirement portfolios rely heavily on equities and bonds to yield a sufficient amount of capital such that the living expenses of retirees can be covered after retirement.

To date, looking at the pension accumulation stage, various pension strategies have been proposed by pension practitioners and scholars. For example, the classical lifecycle pension concept is shown to be an effective and simple pension investment strategy (Fisch and Turner, 2018), which has been used as the default portfolio investment option in the 401(k) plans in the United States (Kilgour, 2019; Chan et al., 2017). Moreover, a dedicated adjustment to the lifecycle pension concept, such as combining it with a liability-based asset allocation component (Crook, 2019) or a target volatility mechanism (Bai and Wallbaum, 2020), can also further improve various pension outcomes required by pension investors.

On the other hand, a significant literature is devoted to improving retirement outcomes in pension decumulation phase (i.e., when investors retire and start making withdrawals from their retirement portfolio). A general consensus is that different retirement outcomes may be achieved by changing the asset allocation (Liu et al., 2019; Louton et al., 2015) or adjusting the annual withdrawal rate (DeJong Jr et al., 2017; Blanchett, 2017; Clare et al., 2017). It has been shown that a retirement portfolio with constant asset allocation can provide attractive portfolio returns (Milevsky et al., 1997; Ho et al., 1994). For example, a retirement portfolio with a risky asset allocation of between 50% and 75% often yields a desirable portfolio return (Bengen, 1994). A fixed asset allocation of 60% equities and 40% bonds is shown to be a suitable asset allocation option for retirement portfolio under different market conditions (Estrada, 2016). In terms of the annual withdrawal rate, which is another important factor that determines the length of coverage of retirement portfolios, a 4% constant annual withdrawal rate is shown to be a fruitful annual withdrawal amount in different market contexts (Bengen, 1994). Nevertheless, all these parameters heavily depend on market environment and interest rate levels.

In an attempt to control the risk that a retirement portfolio may encounter in volatile market conditions, portfolio investment strategies with dynamic asset management components, such as the constant proportion portfolio insurance (CPPI) (Biedova and Steblovskaya, 2020; Temocin et al., 2018; Bernard and Kwak, 2016) and the target volatility concept (Albeverio et al., 2018), have gained important places in pension markets. Lately, a growing body of literature has shown that volatility targeting does help reduce downside risks and add value to retirement investment vehicles over a long-term investment period (Dachraoui, 2018; Perchet et al., 2015). For example, combining a target volatility asset management component with an existing lifecycle investment strategy may improve portfolio returns and reduce overall portfolio risks (Bai and Wallbaum, 2020). Also, volatility targeting might effectively reduce exposure to portfolio tail risk without dragging down portfolio returns (Hocquard et al., 2013).

In this paper, we examine whether the target volatility concept can further improve a retirement portfolio in different aspects of the pension decumulation stage. We consider a target volatility strategy with dynamic target volatility levels linked to interest rates, and we attach the target volatility strategy to retirement portfolios with constant asset allocations as an additional investment protection layer.

Our analysis is based on numerical simulations. Considering the recent unconventional negative interest rate policies adopted in the European marketplace and the overall low interest rate condition worldwide (Wu and Xia, 2020), we choose the suitable financial model and set up our simulation in a way which allows us to study the performance of the newly proposed strategy under a zero or a negative interest rate environment. Also, we evaluate our strategy based on historical time series data extracted from different data sources.

This paper is organized as follows: section 3.2 introduces the dynamic target volatility strategy. Section 3.3 and 3.4 discuss our analysis approach on simulated paths and historical market data, respectively. Section 3.5 and 3.6 present our results. Section 3.7 discusses our findings and concludes this paper.

3.2 The Dynamic Target Volatility Strategy

Market interest rate is usually a good indicator for the movement of economy since the central banks usually cut the interest rate to stimulate consumer spending in a declining economy and raise the interest rate to control inflation in a growing economy (Rochon and Setterfield, 2008). In this paper, we adopted a target volatility strategy with interest rate dependent target volatility levels (addressed as dynamic target volatility strategy). Under the dynamic target volatility strategy, the risky and risk-free asset allocations of a portfolio are determined as follow.

Let T_t and V_t denote the interest rate dependent target volatility level and the realized market volatility at time t, respectively. let L stand for the maximum leverage ratio, and let α_t and β_t represent the risky and risk-free asset allocation of the target volatility portfolio, respectively. The risky and risk-free asset allocations of a target volatility portfolio at time t are determined as follows:

$$\alpha_t = min\{\frac{T_t}{V_t};L\} \tag{3.1}$$

$$\beta_t = 1 - \alpha_t \tag{3.2}$$

The target volatility level T_t at time t is determined with respect to the market interest rate. For example, Albeverio et al. (2018) use the following setting:

$$T_t = \begin{cases} VT_1 \;\; if \;\; r_t < \chi \\ VT_2 \;\; if \;\; r_t \geq \chi \end{cases} \tag{3.3}$$

Here, r_t represents the market interest rate at time t and χ is a pre-specified interest rate trigger, or the interest rate threshold which is used to adjust the target volatility level. The target volatility level equals VT_1 if the market interest rate r_t is smaller than the interest rate trigger χ and VT_2 otherwise.

In this study, we extend the setup of Albeverio et al. (2018) according to our needs (see section 3.3.2).

3.3 Analysis on Simulated Paths

In this paper, we simulate a thirty-year retirement span. At the beginning of the retirement span, the retiree starts marking annual withdrawals from a retirement portfolio of 1 million dollars. We simulate 10,000 return paths for the risky and risk-free assets of the benchmark portfolio and the target volatility portfolio considered. Then, we compute different risk-return measurements to compare the performance of the two portfolios.

This section introduces the financial market model, the portfolio setup, the simulation environment under which we analyze the performance of the target volatility portfolio and the benchmark portfolio, and the risk-return measures for the portfolio performance comparison.

3.3.1 The financial market model

In this section, we introduce the financial market model we use to simulate the returns of embedded assets (i.e., equities and bonds) in the benchmark portfolio and the target volatility portfolio. Specifically, we use the "Hybrid Heston-Vasiček model" adopted in the study by Albeverio et al. (2018) to simulate the returns of risky assets. Let $r(t)$ and $\sigma(t)$ denote the stochastic annual interest rate and stochastic annual volatility, respectively. Under the Hybrid Heston-Vasiček model, the risky asset dynamics $S(t)$ is described by the following stochastic differential equation:

$$dS(t) = r(t)S(t)dt + \sigma(t)S(t)dW_s(t) \tag{3.4}$$

where $r(t)$ follows the Vasiček model (Vasicek, 1977) and $\sigma(t)$ follows the Heston model (Heston, 1993) as follows:

$$dr(t) = k(\theta - r(t))dt + \sigma_r dW_r(t) \tag{3.5}$$

$$d\sigma(t)^2 = v(\beta - \sigma(t)^2)dt + \sigma_\sigma \sigma(t)dW_\sigma(t) \tag{3.6}$$

In equation (3.5), k stands for the speed of reversion of the interest rate, θ is the long-term interest rate reversion level, and σ_r is the volatility of interest rate. The Vasiček model allows for negative interest rates, which is essential for us to study the performance of our newly proposed strategy under a low or a negative interest rate environment. In equation (3.6), v represents the long-term reversion speed of the squared volatility, β is the long-term reversion level of the squared volatility, and σ_σ is the volatility of volatility. The three Wiener processes $W_s(t)$, $W_r(t)$, and $W_\sigma(t)$ are assumed to be correlated as follows:

$$corr(dW_\sigma(t), dW_s(t)) = \rho_{\sigma,s} \tag{3.7}$$

$$corr(dW_s(t), dW_r(t)) = \rho_{s,r} \tag{3.8}$$

$$corr(dW_\sigma(t), dW_r(t)) = \rho_{\sigma,r} = \rho_{\sigma,s}\rho_{s,r} \tag{3.9}$$

We then use the following differential equation with an interest rate component to describe the risk-free asset dynamics:

$$dB(t) = r(t)B(t)dt \tag{3.10}$$

Following Albeverio et al. (2018), we use the following parameter values documented in Huynh et al. (2011) to simulate returns of the risky and risk-free assets:

$$k = 1.25; \theta = 0.05; \sigma_r = 0.025; v = 1.25; \beta = 0.04;$$
$$\sigma_\sigma = 0.2; \rho_{\sigma,s} = -0.5; \rho_{s,r} = 0.2; \rho_{\sigma,r} = 0.1; dt = 1/252.$$

In the main analysis, we begin our simulation using an initial annual interest rate $r(0) = 2\%$ and an initial risky-asset annual volatility $\sigma(0) = 10\%, 15\%, 20\%, 25\%$, and 30%. Given the current low interest rate in different marketplaces (Rubio and Yao, 2020), such as the negative interest rates in the European area, we also change the initial annual interest rate to $r(0) = 0\%$ and compare the results. The goal is to examine the performance of the benchmark portfolio and target volatility portfolio in a low interest rate environment.

3.3.2 Portfolio setup

The benchmark portfolio

To examine whether the target volatility strategy might be a suitable asset management measure when a retirement portfolio enters the pension decumulation phase, we compare the performance of a target volatility portfolio with that of a benchmark portfolio. Early studies have suggested that a retirement portfolio with a simple constant asset allocation can provide attractive portfolio returns (Milevsky et al., 1997; Ho et al., 1994). For example, a retirement portfolio with a risky asset allocation of 50% to 75% often shows good results (Bengen, 1994). Also, a fixed asset allocation of 60% equity and 40% bond can be a suitable asset allocation strategy for a retirement portfolio under different market conditions (Estrada, 2016). Thus, we consider a benchmark portfolio with a constant risky/risk-free asset allocation (i.e., equities/bonds) in a hypothetical thirty-year retirement span. Specifically, we look at five risky/risk-free asset allocation scenarios (i.e., equities/bonds, %): 30/70, 40/60, 50/50, 60/40, and 70/30. We believe that comparing the performance of the target volatility strategy under different asset allocation scenarios can provide more comprehensive results.

The target volatility portfolio

The target volatility portfolio considered is a retirement portfolio that consists of risky assets and risk-free assets. In contrast with the benchmark portfolio, which uses equities and bonds as risky and risk-free assets, respectively, the risky assets in the target volatility portfolio are represented by a benchmark portfolio with the constant asset allocations. The risk-free assets of the target volatility portfolio are local money market instruments and their returns are represented by the simulated bond returns. In this paper, extending Albeverio et al. (2018), we adopt a target volatility strategy with dynamic target volatility levels dependent on market interest rates. Here, we introduce our specific setup for the target volatility strategy.

We consider two interest rate triggers (χ_1 and χ_2) which we use to choose the target volatility level T_t at time t as follows:

$$T_t = \begin{cases} VT_0 \;\; if \;\; \chi_1 \leq r_t \leq \chi_2 \\ VT_1 \;\; if \;\; r_t < \chi_1 \\ VT_2 \;\; if \;\; r_t > \chi_2 \end{cases} \tag{3.11}$$

To determine the initial target volatility level VT_0, we first simulate the return paths of the benchmark portfolio 10,000 times. Then, we compute the average annualized volatilities of the benchmark portfolios and use them as the values of the initial target volatility level VT_0. The idea is to let both the benchmark portfolio and the target volatility portfolio start at the same risk level. We choose a higher target volatility level ($VT_1 = VT_0 + 2\%$) when the historical interest rate drops below the interest rate trigger $\chi_1 = 0\%$ and a lower target volatility level ($VT_2 = VT_0 - 2\%$) when the historical interest rate increases above the interest rate trigger $\chi_2 = 4\%$. We choose the 0% interest rate trigger due to the approximately 0% interest rate in the United States and the persistent negative interest rates in the European area over the past several years. Under this setup, the investors could benefit more from borrowing in a low interest-rate environment and investing in risk-free assets in a high interest rate environment. Table 3.1 summarizes the values we choose for equation (3.11) under the five asset allocation scenarios.

Table 3.1: Target volatility levels with corresponding asset allocation scenarios, $\chi_1 = 0\%$, $\chi_2 = 4\%$

Asset allocation scenarios, equities/bonds, %	VT_0, %	VT_1,%	VT_2, %
30/70	5.84	7.84	3.84
40/60	7.78	9.78	5.78
50/50	9.73	11.73	7.73
60/40	11.67	13.67	9.67
70/30	13.61	15.61	11.61

Example let us fix the risky/risk-free asset allocations at 50%/50%. The initial target volatility level of the target volatility portfolio is set at 9.73%. The target volatility level would be adjusted to 11.73% or 7.73% if the historical interest rate dropped below 0% or raised above 4%, respectively.

After determining the target volatility level T_t at time t based on the historical interest rates, we calculate the risky and risk-free asset allocations using equation (3.1) and (3.2) as follows. We compute the realized market volatility V_t based on the daily returns of equity market over the past twenty days from t, and we set the maximum leverage ratio L at 150% to prevent our target volatility portfolio from overexposing to risky assets (Bai and Wallbaum, 2020). The risky asset allocation of the target volatility portfolio at time t is determined by the ratio between the target volatility level T_t and the realized market volatility V_t at time t. Then, the risk-free asset allocation at time t is computed by using one minus the risky asset allocation at time t. Admittedly, there are different parameter values for determining the target volatility levels. For example, in Albeverio et al. (2018), the authors use an interest rate trigger of 3% with the target volatility level adjusted between 10% and 20% or between 40% and 50%. In this paper, we select our parameter values in line with the current low interest rate environment. A future study may use different parameter values and compare the results.

3.3.3 Simulation environment

In this section, we introduce the transaction costs, inflation rate, and annual withdrawal rate we consider for the simulation environment.

Transaction cost

To generate portfolio return while controlling portfolio risk, the target volatility portfolio needs to adjust its asset allocation between risky and risk-free assets on a daily basis.

Therefore, the transaction costs that occur when this adjustment takes place could have a significant influence on the target volatility portfolio. When using a trend-following investment strategy, Hurst et al. (2017) show that there is a negative correlation between market transaction cost and competitions among market makers, and the transaction costs in equity market moved down from 34bps to 6bps from 1880 to 2013. Thus, we consider a 20bps transaction cost, which is the average of the transaction cost between 1880 and 2013. This is also consistent with the study by Clare et al. (2017), which uses a 20bps transaction cost when examining the performance of trend-following strategies. A higher transaction cost would further lower portfolio ending values and provide more conservative estimates. Future research may examine the performance of the target volatility portfolio under different transaction cost scenarios.

Inflation rate

For a long-term retirement income strategy, the likelihood of a portfolio to provide sufficient retirement coverage is related to uncertainties in portfolio performance. Moreover, it also dependents on changing inflation rates. In general, an increase in inflation rates would lower the purchasing power of money. Retirees living in regions with higher inflation rates would require more retirement incomes to maintain the same living standards compared to those in a region with lower inflation rates. In our simulations with the thirty-year retirement horizon, we set an annual inflation rate at 2%, which is the average annual inflation rate in the United States between 2015 and 2020.

Annual withdrawal rate

Settling a proper fixed annual withdrawal rate for optimizing retirement portfolio performance is a subject that has been discussed extensively in the domain of retirement practice. The earliest discussion on the annual withdrawal rate of retirement portfolio can be dated back to the 4% constant rule (Bengen, 1994), which is further confirmed as a suitable choice for retirement planning when having a portfolio of greater than 50% equity in asset allocation (Cooley et al., 2003). Following this path, we evaluate the performance of the target volatility portfolio and benchmark portfolio under a fixed annual withdrawal rate environment. Specifically, we use 3.33% (i.e., 1/length of retirement span) as the fixed annual withdrawal rate. In other words, over the thirty-year retirement span, the retiree makes an annual withdrawal amount that equals to 3.33% of the initial value at the beginning of the thirty-year retirement span.

3.3.4 Risk-return measures on simulated paths

Portfolio durability measures

Average length of the retirement coverage. To measure the durability of the two portfolios considered (i.e., the benchmark portfolio and the target volatility portfolio), we first compute the average length of the retirement coverage (in years): the average number of years it takes for each portfolio to run out of capital within the thirty-year retirement span.

Portfolio survival rate. We then show the portfolio survival rate, which is the percentage of the simulated portfolio paths with positive terminal values. A higher portfolio survival rate indicates that the retirement portfolio has a higher likelihood to outlive the retiree when looking at the thirty-year retirement span.

Portfolio risk measures

95% value-at-risk (VAR). We use the 95% VAR of the portfolio terminal values to measure the portfolio downside risk. Specifically, we order the portfolio terminal values in an ascending order and use the 95% location to approximate the value of VAR. The 95% VAR measures the worst 5% portfolio ending values at the end of the thirty-year retirement span. For example, a 95% VAR of $100,000 suggests that there is a 95% chance that the portfolio terminal value will be higher than $100,000.

95% conditional value-at-risk (CVAR). The 95% CVAR is considered as a more conservative measurement for the portfolio tail risk. As an extended risk measurement of the 95% VAR, it is computed by taking the average of portfolio terminal values below the 5% cutoff point.

Portfolio return measure

Average annualized return. To evaluate the performance of the target volatility portfolio and the benchmark portfolio, we calculate the average annualized return of the portfolio over the retirement span. For the 10,000 simulations, we compute the annualized returns for each simulated path and then take the average of the annualized returns.

3.4 Analysis on Historical Data

To further evaluate the performance of the target volatility portfolio and the benchmark portfolio, we repeat our analysis on the historical time series data of the European market since the European region shows negative interest rates over the past several years. In this section, we introduce the historical market data used for analysis and the risk-return measurements we calculate to compare the performance of the benchmark portfolio and the target volatility portfolio.

3.4.1 Historical Market Data

We use the STOXX Europe 600 Index (Caporin et al., 2021) and the Bloomberg Barclays Euro Aggregate Bond Index (Matsui, 2020) to represent the equity returns and bond returns, respectively. We use the 3-month Internal Bank Rate for Euro Area as a proxy of the interest rate of the European area, and we extract the longest available historical time series from the Bloomberg database and the Federal Reserve Economic Data (FRED) database until the time of our analysis. Subsequently, we end up with approximately 24 years of daily data from 06.30.1998 to 10.21.2021.

3.4.2 Risk-return measures on historical data

Portfolio durability measures

Average length of the retirement coverage. Over the 24-year retirement span, we compute the number of years the two portfolios run out of capital.

Recall that for the analysis on the 10,000 simulated paths, we compute the portfolio survival rate (the percentage of the simulated portfolio paths with positive terminal values). Since the historical analysis is based on a single historical return path, we exclude this measurement from the historical analysis.

Portfolio risk measures

95% value-at-risk (VAR). For the historical analysis, we compute the 95% VAR based on the one-year rolling return of the portfolio. Specifically, we compute the annual return of the portfolio starting from the first financial year. Then we compute the one-year rolling return by rolling the annual return on a daily basis for the 24-year retirement span. We order the one-year rolling returns in an ascending order and use the 95% location to approximate the value of VAR. As a result, the 95% VAR measures the worst 5% portfolio return over an one-year investment window.

95% conditional value-at-risk (CVAR). The 95% CVAR is computed by taking the average of the one-year rolling return below the 5% cutoff point.

Portfolio return measure

Average annual return. Regarding the portfolio return measure, we compute the annual returns of the benchmark portfolio and the target volatility portfolio for each financial year within the 24-year retirement span. Then, we take the average of the annual returns as the average annual return.

3.5 Numerical Results on Simulated Paths

3.5.1 Initial annual interest rate r(0) = 2%

We first report the results of our numerical simulations with the initial annual interest rate $r(0) = 2\%$ (see section 3.3.1). The initial risky asset annual volatility $\sigma(0)$ varies between 10% and 30%. Recall that for each combination of initial interest rate $r(0)$ and initial risky asset volatility $\sigma(0)$, we simulated 10,000 paths according to the market model described in section 3.3.1.

Table 3.2 presents the first durability measure: the average length of the retirement coverage (in years) of the benchmark portfolio and the target volatility portfolio. In general, for all the asset allocation scenarios considered, the target volatility portfolio provides a longer period of effective retirement coverage in contrast with the benchmark portfolio. For example, when the risky/risk-free asset allocations are 70%/30% and the initial risky asset annual volatility $\sigma(0) = 30\%$, the target volatility portfolio offers approximately 2 years longer retirement coverage than the benchmark portfolio.

Table 3.3 presents the results of the second portfolio durability measure: portfolio survival rate (i.e., the percentage of simulations with a positive portfolio terminal value). The most pronounced difference regarding the portfolio survival rate can be seen when the risky/risk-free asset allocations are 70%/30% and the initial risky asset annual volatility $\sigma(0) = 30\%$, where the target volatility portfolio improves the portfolio survival rate by around 23% in contrast with the benchmark portfolio. The differences in the portfolio survival rate (TVP - BP) are all positive, suggesting that the target volatility strategy can improve the portfolio survival rate by 2.88% to 23.28% for the five asset allocation scenarios considered.

Table 3.2: The length of the retirement coverage (in years), initial annual interest rate $r(0) = 2\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP)					
Equities/bonds, %					
30/70	29.92	29.91	29.90	29.89	29.87
40/60	29.67	29.65	29.62	29.59	29.54
50/50	29.24	29.21	29.16	29.09	29.01
60/40	28.67	28.63	28.55	28.47	28.36
70/30	28.03	27.97	27.88	27.77	27.63
Target volatility portfolio (TVP)					
Equities/bonds, %					
30/70	30.00	30.00	30.00	30.00	30.00
40/60	30.00	30.00	30.00	30.00	30.00
50/50	29.99	29.99	30.00	30.00	30.00
60/40	29.88	29.90	29.92	29.93	29.95
70/30	29.46	29.52	29.58	29.64	29.69
Difference (TVP - BP)					
Equities/bonds, %					
30/70	0.08	0.09	0.10	0.11	0.13
40/60	0.33	0.35	0.38	0.41	0.46
50/50	0.75	0.78	0.84	0.90	0.99
60/40	1.21	1.27	1.36	1.46	1.59
70/30	1.43	1.55	1.70	1.88	2.06

Table 3.4 shows the 95% VAR of the benchmark portfolio and target volatility portfolio based on portfolio terminal values. Looking at the hypothetical thirty-year retirement span, the target volatility portfolio shows a significantly higher 95% VAR (between -144.48 and 456.10 thousand dollars) in contrast with the benchmark portfolio (between -462.70 and 63.71 thousand dollars), suggesting that the target volatility portfolio can better control the portfolio downside risk.

We see similar pattern regarding the 95% CVAR presented in Table 3.5. Compared to the benchmark portfolio with a 95% CVAR ranges between -637.21 and -35.60 thousand dollars, the target volatility portfolio shows a 95% CVAR between -254.24 and 403.67 thousand dollars, respectively.

Table 3.6 presents the average annualized return of benchmark portfolio and target volatility portfolio based on the 10,000 simulations. As shown in the table, the average annualized

Table 3.3: Portfolio survival rate, %. Initial annual interest rate $r(0) = 2\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP),					
Equities/bonds, %					
30/70	97.12	96.93	96.77	96.53	95.98
40/60	91.60	91.16	90.77	90.10	89.45
50/50	84.62	84.19	83.44	82.56	81.68
60/40	77.43	77.96	76.54	75.96	74.99
70/30	71.35	70.96	70.32	69.61	68.54
Target volatility portfolio (TVP)					
Equities/bonds, %					
30/70	100.00	100.00	100.00	100.00	100.00
40/60	100.00	100.00	100.00	100.00	100.00
50/50	99.56	99.70	99.85	99.89	99.93
60/40	96.12	96.56	96.99	97.49	97.98
70/30	87.92	88.86	89.99	90.94	91.82
Difference (TVP - BP)					
Equities/bonds, %					
30/70	2.88	3.07	3.23	3.47	4.02
40/60	8.40	8.84	9.23	9.90	10.55
50/50	14.94	15.51	16.41	17.33	18.25
60/40	18.69	19.50	20.45	21.53	22.99
70/30	16.57	17.90	19.67	21.33	23.28

Table 3.4: Portfolio 95% VAR (in thousand dollars). Initial annual interest rate $r(0) = 2\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP)					
Equities/bonds, %					
30/70	63.71	55.61	47.27	39.86	27.19
40/60	-81.41	-90.24	-98.16	-108.70	-124.22
50/50	-200.04	-210.77	-221.55	-235.75	-251.92
60/40	-308.56	-316.04	-329.16	-343.65	-363.60
70/30	-400.78	-407.62	-423.40	-441.88	-462.70
Target volatility portfolio (TVP)					
Equities/bonds, %					
30/70	444.10	447.79	450.51	453.66	456.10
40/60	346.41	354.33	361.89	368.04	373.80
50/50	198.10	210.43	223.50	235.43	247.37
60/40	27.24	42.29	59.72	76.89	94.25
70/30	-144.48	-129.11	-112.15	-93.11	-73.28
Difference (TVP - BP)					
Equities/bonds, %					
30/70	380.39	392.18	403.23	413.80	428.91
40/60	427.82	444.57	460.05	476.74	498.01
50/50	398.14	421.20	445.05	471.19	499.29
60/40	335.79	358.33	388.88	420.54	457.85
70/30	256.30	278.51	311.25	348.77	389.43

Table 3.5: Portfolio 95% CVAR (in thousand dollars). Initial annual interest rate $r(0) = 2\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP)					
Equities/bonds, %					
30/70	-35.60	-41.33	-49.34	-60.05	-73.04
40/60	-185.58	-192.79	-202.92	-216.33	-232.70
50/50	-315.71	-324.78	-337.59	-354.10	-373.96
60/40	-435.06	-446.59	-462.26	-481.26	-504.40
70/30	-553.65	-567.08	-585.69	-609.55	-637.21
Target volatility portfolio (TVP)					
Equities/bonds, %					
30/70	390.17	394.01	397.46	400.67	403.67
40/60	277.40	285.89	294.35	302.45	310.15
50/50	114.63	127.38	141.25	154.87	167.89
60/40	-69.28	-53.53	-35.05	-16.42	1.56
70/30	-254.24	-236.92	-214.91	-192.00	-169.63
Difference (TVP - BP)					
Equities/bonds, %					
30/70	425.77	435.34	446.80	460.72	476.71
40/60	462.98	478.68	497.26	518.78	542.86
50/50	430.34	452.16	478.84	508.97	541.85
60/40	365.78	393.06	427.21	464.85	505.96
70/30	299.41	330.16	370.77	417.55	467.58

Table 3.6: Average annualized return, %. Initial annual interest rate $r(0) = 2\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP)					
Equities/bonds, %					
30/70	4.445	4.445	4.445	4.446	4.446
40/60	4.463	4.463	4.464	4.464	4.465
50/50	4.482	4.483	4.484	4.484	4.485
60/40	4.503	4.504	4.505	4.506	4.506
70/30	4.524	4.526	4.527	4.528	4.529
Target volatility portfolio (TVP)					
Equities/bonds, %					
30/70	4.383	4.388	4.391	4.393	4.395
40/60	4.384	4.389	4.393	4.396	4.397
50/50	4.392	4.395	4.399	4.402	4.404
60/40	4.415	4.407	4.410	4.412	4.414
70/30	4.458	4.430	4.427	4.429	4.430
Difference (TVP - BP)					
Equities/bonds, %					
30/70	-0.061	-0.057	-0.054	-0.052	-0.051
40/60	-0.079	-0.074	-0.071	-0.069	-0.067
50/50	-0.090	-0.088	-0.085	-0.083	-0.081
60/40	-0.088	-0.097	-0.095	-0.093	-0.092
70/30	-0.066	-0.096	-0.100	-0.101	-0.099

return of the benchmark portfolio are slightly higher than that of the target volatility portfolio under all the asset allocation scenarios studied. For example, the target volatility portfolio shows an 0.101% lower average annualized return in contrast with the benchmark portfolio when the risky/risk-free asset allocations are 70%/30% and the initial risky asset annual volatility $\sigma(0) = 25\%$. This is also the most significant difference regarding the average annualized return between the target volatility portfolio and the benchmark portfolio in our analysis. As we will discuss in section 3.6, this difference regarding the average annualized return should not be a barrier that shies away the benefits of the target volatility portfolio.

3.5.2 Initial annual interest rate r(0) = 0%

Over the past several years, we see that the negative interest rate policy has been used to stimulate economic growth in the European area. For example, the most recent 3-month London Interbank Offered Rate (LIBOR) is around -0.57% based on data from the Federal Reserve Economic Data (FRED) database. This unconventional measure casts new doubt on the investment portfolios that rely on risk-free assets such as bonds. Therefore, studying the performance of a long-term investment portfolio under an zero or a negative interest rate environment is necessary. Here, we repeat our simulations with an initial annual interest $r(0) = 0\%$. Since we change the initial annual interest rate to a lower level, we also update the interest rate triggers to -2% and 2% (i.e., $\chi_1 = -2\%$ and $\chi_2 = 2\%$) for choosing the target volatility level.

Table 3.7 presents the summary of the quantitative comparison on the portfolio durability, risk, and return measures when the initial annual interest rate $r(0) = 0\%$. Comprehensive results based on the initial interest rate $r(0) = 0\%$ can be found in Appendix A for Chapter 3 in the end of this chapter. Here, we see similar tendencies to the results from the initial annual interest rate $r(0) = 2\%$ where the target volatility portfolio provides more durable

Table 3.7: Quantitative comparison on the portfolio durability, risk, and return measures: the target volatility portfolio measures minus the benchmark portfolio measures (TVP - BP). Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Length of retirement coverage (in years)					
Equities/bonds, %					
30/70	0.10	0.11	0.12	0.13	0.15
40/60	0.38	0.40	0.43	0.47	0.51
50/50	0.82	0.86	0.91	0.99	1.07
60/40	1.27	1.34	1.44	1.55	1.68
70/30	1.47	1.60	1.76	1.94	2.14
Portfolio survival rate, %					
Equities/bonds, %					
30/70	3.51	3.59	3.81	4.22	4.62
40/60	9.38	9.80	10.30	10.85	11.55
50/50	16.19	16.73	17.60	18.49	19.32
60/40	18.83	19.93	21.18	22.34	23.76
70/30	16.76	18.13	19.63	21.48	23.49
95% VAR (in thousand dollars)					
Equities/bonds, %					
30/70	371.98	384.16	394.75	406.17	418.79
40/60	418.37	433.15	450.24	467.54	487.45
50/50	388.39	412.86	437.19	461.77	488.90
60/40	326.44	349.80	380.15	413.52	448.25
70/30	250.34	272.74	308.36	346.41	384.88
95% CVAR (in thousand dollars)					
Equities/bonds, %					
30/70	416.54	426.05	437.39	451.25	467.11
40/60	453.46	469.14	487.74	509.15	533.06
50/50	422.52	444.45	471.10	501.13	533.83
60/40	360.61	388.36	422.27	459.94	501.18
70/30	296.73	328.64	369.37	416.03	465.97
Average annualized return, %					
Equities/bonds, %					
30/70	-0.06	-0.055	-0.052	-0.050	-0.049
40/60	-0.077	-0.072	-0.068	-0.066	-0.065
50/50	-0.088	-0.086	-0.082	-0.080	-0.078
60/40	-0.089	-0.095	-0.092	-0.090	-0.089
70/30	-0.073	-0.095	-0.098	-0.097	-0.096

retirement coverage, lower downside risk, but lower average portfolio returns in contrast with the benchmark portfolio. Another important pattern we find is that the difference between the target volatility portfolio and the benchmark portfolio regarding the two durability measurements are more pronounced when the initial annual interest rate $r(0) = 0\%$ than that when the initial annual interest rate $r(0) = 2\%$. Let us fix the risky/risk-free asset allocations to 50%/50% and the initial risky asset annual volatility $\sigma = 20\%$. The target volatility portfolio provides a 0.91 years longer retirement coverage compared to the benchmark portfolio when the initial annual interest rate is $r(0) = 0\%$ in contrast with an extended retirement coverage of 0.84 years when the initial annual interest rate is $r(0) = 2\%$. Under the same risky/risk-free asset allocations and initial risky asset annual volatility, the target volatility can improve the portfolio survival rate by 17.60% and 16.41% when the initial annual interest rates $r(0) = 0\%$ and $r(0) = 2\%$, respectively. This finding provides evidence that the benefits of target volatility strategy is even more significant under the zero and the negative interest rate environment.

3.6 Numerical Results on Historical Data

In Table 3.8, we provide a summary for the portfolio durability, risk, and return measurements based on historical data. Regarding the length of retirement coverage, we see that both portfolios are capable of providing a full 24-year retirement coverage when the risky asset allocation ranges from 30% to 70%. Nonetheless, when the risky/risk-free asset allocations are 70%/30%, the target volatility portfolio offers 1 additional year of retirement coverage in contrast with the benchmark portfolio.

Regarding the 95% VAR and CVAR based on the one-year rolling return, the most significant difference regarding the 95% VAR can be found when the risky/risk-free asset allocations are 50%/50% where the target volatility portfolio has 6.15% higher 95% VAR than the benchmark portfolio. We see similar results for the 95% CVAR. The target volatil-

Table 3.8: The portfolio durability, risk, and return measures from historical analysis

	Benchmark portfolio (BP)	Target volatility portfolio (TVP)	Difference (TVP - BP)
Length of retirement coverage (in years)			
Equities/bonds, %			
30/70	24	24	0
40/60	24	24	0
50/50	24	24	0
60/40	24	24	0
70/30	23	24	1
95% VAR, %			
Equities/bonds, %			
30/70	-5.03	-1.76	3.27
40/60	-8.54	-3.66	4.88
50/50	-12.19	-6.03	6.15
60/40	-15.57	-9.79	5.78
70/30	-18.82	-14.23	4.59
95% CVAR, %			
Equities/bonds, %			
30/70	-8.69	-3.06	5.63
40/60	-12.85	-5.09	7.77
50/50	-16.94	-7.83	9.11
60/40	-20.87	-11.53	9.35
70/30	-24.64	-16.66	7.97
Average annual return, %			
Equities/bonds, %			
30/70	4.34	3.47	-0.87
40/60	4.35	3.17	-1.18
50/50	4.40	3.09	-1.31
60/40	4.49	3.29	-1.20
70/30	4.62	3.61	-1.01

ity portfolio outperforms the benchmark portfolio by 9.11% and 9.35% regarding the 95% CVAR when the risky/risk-free asset allocations are 50%/50% and 60%/40%, respectively.

Results for the return measurement: average annual returns are similar to those from the simulation approach. The target volatility portfolio underperforms the benchmark portfolio by 0.87% to 1.31% for the five risky/risk-free asset allocation scenarios studied in this paper.

We do not report the portfolio survival rate (the percentage of simulated paths with positive terminal values), since this part of analysis is based on a single historical return path.

3.7 Discussion and Conclusion

In this paper, we use the target volatility strategy to enhance the retirement coverage of conventional retirement portfolios with constant risky/risk-free asset allocations. We consider a dynamic target volatility strategy under which the target volatility levels are linked to market interest rates. We then evaluate the performance of the newly proposed strategy using both a simulation approach over a 30-year retirement span and a historical analysis approach under a 24-year retirement span.

Using the simulated data for low interest rate market environments (initial annual interest rate $r(0) = 2\%$ and $r(0) = 0\%$), we find that the target volatility portfolio can enhance the length of the retirement coverage by more than 2 years and improve the portfolio survival rate by more than 20% compared to the benchmark portfolio with constant asset allocations. We find similar pattern on historical market data where the target volatility portfolio can provide an additional year of retirement coverage compared to the benchmark portfolio. This finding is rather important considering the low interest rate market environment globally and the persistent negative interest rates in the European area.

On both simulated data and historical market data, our numerical results regarding the 95% VAR and CVAR suggest that the target volatility portfolio can better control portfolio downside risks compared to the benchmark portfolio.

Our numerical results also show that the target volatility portfolio provides lower average annualized returns compared to the benchmark portfolio. This finding can be attributed to the asset classes of the target volatility portfolio. Recall that in our construction of the portfolio, the benchmark portfolio is used as a risky asset. Subsequently, the overall equity allocation of the target volatility portfolio is lower than that of the benchmark portfolio. Nonetheless, the lower total portfolio return of the target volatility portfolio should not be an obstacle that shies away the benefits of the target volatility strategy. For example, based on the results from our simulation approach (reported in Table 6), the most significant difference on the average annualized return is -0.101% when the risky/risk-free asset allocations in the benchmark portfolio are 70%/30% and the initial risky asset annual volatility $\sigma(0) = 25\%$. Under the same asset allocation and risky asset annual volatility scenario, the target volatility portfolio improves the length of retirement coverage by 1.88 years and the portfolio survival rate by 21.33%. Thus, the negative difference in the average annualized return is marginal and may not significantly affect the quality of the retirement coverage for retirees.

Several reasons help explain why the target volatility portfolio has more potential enhancing the retirement coverage compared to a conventional retirement portfolio. First, the target volatility strategy is a dynamic asset management strategy that adjusts the asset allocation of a portfolio periodically. When the market risk is above the target volatility level, the target volatility portfolio allocates more capital to risk-free assets and vice versa. This allows the target volatility portfolio to control the portfolio risk under the volatile market conditions. Second, one important mechanism in the dynamic target volatility strategy is that it increases (decreases) the target volatility level when the interest rate drops below (increases above) the specified threshold. In other words, the target volatility portfolio may allocate more capital into the risky asset by increasing its target volatility level when the market interest rate decreases. This mechanism allows investors to benefit more from borrowing in a zero or a negative interest rate environment.

In this paper, we focus on using the dynamic target volatility strategy to improve the conventional retirement plans with simple asset allocations (equities and bonds) in the pension decumulation stage. In a future work, we plan to study using the target volatility investment concept to improve a pension plan with more complex asset allocations (e.g., the constant proportion portfolio insurance (CPPI) with a minimum return guarantee, refer to Biedova and Steblovskaya (2020) for details). Following the dynamic target volatility strategy examined in this paper, another important subject might be studying the optimal interest rate thresholds for the target volatility strategy to derive desirable retirement outcomes. Moreover, since we consider a fixed retirement span, how to mitigate the longevity risk using the target volatility investment concept may also merit further exploration.

Reducing Transaction Costs: The Optimal Rebalancing Boundary For Target Volatility Strategy In Different Market Conditions

4.1 Introduction

Transaction costs are costs that arise when making business decisions (Kissell, 2006). The target volatility strategy is an asset management concept which aims at maintaining portfolio risk in volatile market conditions (Albeverio et al., 2013, 2018, 2019). This is achieved by adjusting the asset allocation of a portfolio on a regular basis. Thus, we define transaction costs as the costs accumulated when the target volatility strategy adjusts its risky and risk-free asset allocation over an investment horizon.

Transaction costs might affect the performance of an investment strategy from two different aspects. First, for an asset management strategy embedded with a dynamic asset allocation mechanism, transaction costs may lower portfolio returns directly (Jana et al., 2009). Second, the presence of high transaction costs might make investors to switch from an active asset reallocation strategy (e.g., target volatility strategy) to a passive asset reallocation strategy (e.g., a constant asset allocation strategy) (Rowland, 1999). Moreover, looking at cross-country portfolio investments, markets where transaction costs are low often attract more investments than markets where transaction costs are high (Thapa and Poshakwale, 2010). Thus, how to reduce the deleterious effect of transaction cost on portfolio return is an important step to achieve desirable portfolio outcomes.

To address the influence of transaction costs on asset management strategies, some studies have incorporated transaction costs in an attempt to further improve an existing portfolio optimization model (see Yoshimoto (1996); Konno and Wijayanayake (2001); Atkinson and Mokkhavesa (2004); Babazadeh and Esfahanipour (2019) for examples). Others add transaction costs as a crucial factor when analyzing asset management strategies and market risks (e.g., Li et al. (2019); Zakamulin (2019)). In this paper, we attempted to reduce the transaction cost that a target volatility strategy may encounter by adding a rebalancing boundary to its volatility target asset allocation mechanism. To do so, we first proposed a constraint optimization algorithm where we identified the optimal rebalancing boundary level that maximizes the portfolio return measure (Omega ratio) while controls its risk measure (mean volatility deviation) under a given threshold. We implement the optimization algorithm under two different simulation environment: 1) the stylized Black-Scholes environment where market return and market volatility are assumed at a constant level; 2) the bootstrapped environment where market returns are randomly sampled based on real historical data. Then, we evaluate the performance of the target volatility portfolio with the optimal rebalancing boundary (called Optimized Portfolio E) in contrast with a traditional target volatility portfolio without such rebalancing boundaries (called Portfolio S) by conducting comparative analysis under different market scenarios. To the best of our knowledge, our paper is the first attempt to address the problem of minimizing transaction costs in the context of target volatility strategies.

The rest of this paper is organized as follows: section 4.2 introduces the asset allocation mechanism of Portfolio S and Portfolio E. In section 4.3, we present the risk and the return measurements to formulate the constraint optimization problem. Section 4.4 introduces the numerical optimization algorithm. Section 4.5 and 4.6 present the implementation of the optimization algorithm within the Black-Scholes environment and the bootstrapped environment, respectively. In section 4.7, we evaluate the performance of Portfolio S and Optimized Portfolio E under different market scenarios. Section 4.8 concludes this paper.

4.2 Target Volatility Strategies

We consider two target volatility portfolios: a benchmark target volatility portfolio (Portfolio S) and a target volatility portfolio with rebalancing boundaries (Portfolio E). Both Portfolio S and Portfolio E contain a risky asset (represented by equities) and a risk-free asset (represented by bonds).

4.2.1 Benchmark Target Volatility Portfolio S

Let T denote the target volatility level, L denote the maximum leverage ratio, and V_t represent the realized annualized volatility of a risky asset at time t. The risky and risk-free asset allocations (α_t and β_t, respectively) of a benchmark target volatility Portfolio S at time t are defined as follows:

$$\alpha_t = min\{\frac{T}{V_t}; L\} \tag{4.1}$$

$$\beta_t = 1 - \alpha_t \tag{4.2}$$

4.2.2 The Target Volatility Portfolio E with Rebalancing Boundaries

Let us denote by ϕ a rebalancing boundary which we will add to the benchmark portfolio rebalancing protocol. The target volatility portfolio E with the rebalancing boundary ϕ only adjusts its asset allocation when the ratio $\frac{T}{V_t}$ deviates from the ratio $\frac{T}{V_{t-1}}$ in absolute value by at least ϕ:

$$\alpha_t = \begin{cases} min\{\frac{T}{V_t}; L\}, & if \ \ |\frac{T}{V_t} - \frac{T}{V_{t-1}}| > \phi \\ \alpha_{t-1} & if \ \ |\frac{T}{V_t} - \frac{T}{V_{t-1}}| \leq \phi \end{cases} \tag{4.3}$$

$$\beta_t = 1 - \alpha_t. \tag{4.4}$$

For example, let us consider target volatility Portfolios S and E based on the same risky and risk-free assets with the following parameters: both portfolios have 12% target volatility level ($T = 0.12$) and 150% maximum leverage ratio ($L = 1.5$). Portfolio E has a rebalancing boundary of 4% ($\phi = 0.04$). On the first trading day, the realized annual risky asset volatility was 20% ($V_t = 0.20$).

$$\alpha_1 = min\{\frac{0.12}{0.20}; 1.50\} = 0.60$$
$$\beta_1 = 1 - 0.60 = 0.40$$

Both target volatility portfolios would allocate 60% of the capital into risky assets and the remaining 40% into risk-free assets. On the next trading day, suppose the realized annual risky asset volatility changed to 19% ($V_{t+1} = 0.19$).

$$\alpha_2 = min\{\frac{0.12}{0.19}; 1.50\} = 0.6316$$
$$\beta_2 = 1 - 0.6316 = 0.3684$$

The benchmark Portfolio S would allocate 63.16% of the capital into the risky asset and the remaining 36.84% into the risk-free asset.

The target volatility Portfolio E with a 4% rebalancing boundary would maintain the same asset allocation as the one on the prior trading day (i.e., 60% risky assets and 40% risk-free assets) until the ratio of the target volatility to the realized risky asset volatility exceeded $60\% + 4\% = 64\%$ or dropped below $60\% - 4\% = 56\%$.

On the third trading day, suppose the realized annual risky asset volatility level changed to

18%.

$$\alpha_3 = min\{\frac{0.12}{0.18};1.50\} = 0.6667$$

$$\beta_3 = 1 - 0.6667 = 0.3333$$

Then, both portfolios would allocate 66.67% of the capital into risky assets and the remaining 33.33% into risk-free assets since the ratio of the target volatility to the realized risky asset volatility exceeded the upper limit of the rebalancing boundary (i.e., 64%).

This mechanism allows us to save some transaction costs as will be illustrated in the next sections.

Figure 4.1 shows a comparison of the asset allocation with and without a 3% rebalancing boundary. As shown in the highlighted section, the rebalancing boundary allows the target

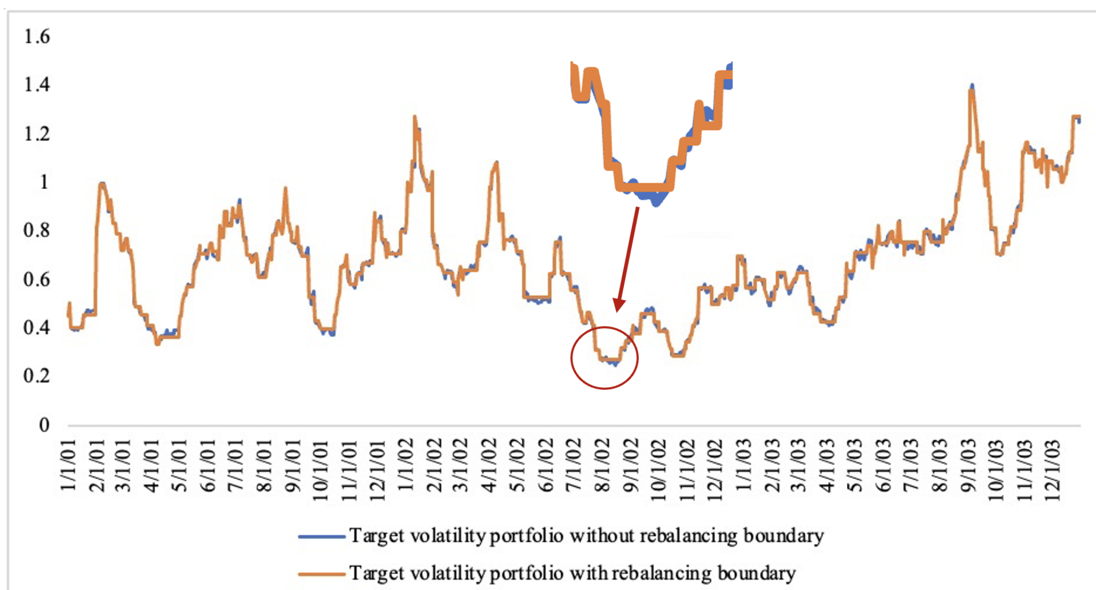


Figure 4.1: An illustration of asset allocation with rebalancing boundary

volatility portfolio to maintain the asset allocation at a constant level over some investment timeframe while the portfolio without the rebalancing boundary adjusts its asset allocation on a daily basis.

4.3 Portfolio performance measures

In this section, we introduce the portfolio performance measures that will be used for our proposed numerical optimization algorithm.

4.3.1 The risk measure

As a risk measure of a portfolio, we will use the average volatility deviation from the target volatility T. More specifically. Let us consider an investment period $[0,\tau]$. We will split it into n subintervals $[t_k,t_{k+1})$, $k=0,\dots,n-1$, $t_0=0, t_n=\tau$. Denote by $V^S_{t_k}$ and $V^{E,\phi}_{t_k}$ the annualized portfolio volatility of Portfolio S and Portfolio E, respectively, at time t_k. Here ϕ is a rebalancing boundary for Portfolio E. Further, denote by $D^S_{t_k}$ (resp. $D^{E,\phi}_{t_k}$) the volatility deviation from the target volatility T for Portfolio S (resp. Portfolio E) at time t_k $k=1,\dots,n$:

$$D^S_{t_k} = max(V^S_{t_k} - T, 0) \tag{4.5}$$

$$D^{E,\phi}_{t_k} = max(V^{E,\phi}_{t_k} - T, 0) \tag{4.6}$$

Then the average volatility deviation from the target volatility T for portfolios S and E is defined as follows:

$$D^S = \frac{1}{n}\sum_{k=1}^{n} D^S_{t_k} \tag{4.7}$$

$$D^{E,\phi} = \frac{1}{n}\sum_{k=1}^{n} D^{E,\phi}_{t_k} \tag{4.8}$$

4.3.2 The return measure

We use the Omega ratio (Keating and Shadwick, 2002) as the return measure of a portfolio. Let X be a portfolio percentage return over a certain period of time. Suppose X has the cumulative probability distribution function F. Denote by a a pre-specified loss threshold. Then the Omega ratio is defined as follows:

$$\Omega_X(a) = \frac{\int_a^\infty [1-F(x)]dx}{\int_{-\infty}^a F(x)dx} \tag{4.9}$$

Alternatively, the Omega ratio can be computed as a ratio between the expected return above a given threshold and the expected loss below a given threshold (Kazemi, 2012) as follows:

$$\Omega_X(a) = \frac{E[max(X-a,0)]}{E[max(a-X,0)]}. \tag{4.10}$$

Let X^S (resp. X^E) denote the Portfolio S (resp. Portfolio E) percentage return over the investment period $[0,\tau]$:

$$X^S = \frac{P_\tau^S - P_0^S}{P_0^S} \tag{4.11}$$

$$X^E = \frac{P_\tau^E - P_0^E}{P_0^E}, \tag{4.12}$$

where P_t^S (resp. P_t^E) stands for the Portfolio S (resp. Portfolio E) value at time t. We will denote the Omega ratio (with the threshold a) of the benchmark Portfolio S by $\Omega_S(a)$. Let ϕ be the rebalancing boundary of Portfolio E. We will denote the Omega ratio (with the threshold a) of Portfolio E with the rebalancing boundary ϕ by $\Omega_{E,\phi}(a)$.

Notice that both $\Omega_S(a)$ and $\Omega_{E,\phi}(a)$ take into account transaction costs.

4.4 Portfolio optimization problem and numerical optimization algorithm

In this section, we propose a numerical optimization algorithm which allows an investor to chose the best rebalancing boundary for Portfolio E according to an investor-relevant optimization criterion.

4.4.1 Optimization problem formulation

Over an investment period $[0,\tau]$, let us consider a benchmark target volatility Portfolio S with parameters T and L as in Section 4.2 and Portfolio E based on the same risky and risk free assets as Portfolio S and with the same parameters T and L. Let A denote a preliminary set of rebalancing boundaries for Portfolio E and let a denote a pre-specified loss threshold. We also specify the transaction costs arrangements.

The constrained optimization problem is formulated as follows:

$$Maximize\ \Omega_{E,\phi}(a) \tag{4.13}$$

$$subject\ to\ D^{E,\phi} \leq D^S \tag{4.14}$$

$$Over\ \phi \in A$$

4.4.2 Numerical optimization algorithm

We propose a numerical optimization algorithm to solve the above optimization problem.

Input parameters: $[0,\tau]$, T, L, a, set A, transaction costs level(s).

1. Using historical data, simulate 10,000 risky asset paths over the future investment period $[0,\tau]$;

2. Simulate the performance of Portfolio S and Portfolio E (for each ϕ in A) on each risky asset path including transaction costs;

3. Estimate the Portfolio S volatility deviation D^S and the Portfolio E volatility deviation $D^{E,\phi}$ for each $\phi \in A$ by averaging the corresponding volatility deviations over simulated paths.

4. Determine the admissible set of rebalancing boundaries $B \subset A$ such that for each $\phi \in B$, $D^{E,\phi} \leq D^S$.

5. For each rebalancing boundary level $\phi \in B$, calculate the Omega ratio $\Omega_{E,\phi}(a)$ using the average returns above and below the threshold a over all simulated paths (including transaction costs).

6. Find the optimal rebalancing boundary level ϕ^* that maximizes $\Omega_{E,\phi}(a)$.

As an attempt to control portfolio risk, the accumulated transaction cost from adjusting the risky and risk-free allocations of the target volatility portfolio might be a crucial factor affecting portfolio returns. Thus, we follow Di Persio et al. (2021) and use volatility-linked transaction costs for our analysis as follows:

Table 4.1: Market volatility levels and transaction costs

Market volatility level	Transaction cost
Below 10%	10 basis points (bps)
Between 10% to 30%	20 basis points (bps)
More than 30%	50 basis points (bps)

4.5 Illustrative numerical experiments within the Black-Scholes Environment

4.5.1 Financial model and parameter values for simulation

In our stylized environment, the market follows the Black-Scholes model. In other words, we assume that the risky asset dynamics is described as follows:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{4.15}$$

where μ and σ denote the expected annual return and the annual volatility of the risky asset, respectively. W_t is the standard Wiener process. The risk-free asset (bond) grows at a constant annual rate r.

We consider the following parameter input values: $\mu = 0.07$ and $\sigma = 0.20, 0.30, 0.40$, and we use a constant annual interest rate $r = 0.02$.

4.5.2 Portfolio setup

For our simulations we use the following parameters: $\tau = 30$, $T = 0.12$, $L = 1.50$, $r = 0.02$, $a = 0, 0.01, 0.02, 0.03, 0.04$, $A = \{0.005, 0.01, 0.015, 0.02, 0.025\}$. We estimate the annualized market volatility V_t daily using the standard deviations of the daily returns over the past 20 days before t. We rebalance Portfolio S daily and Portfolio E according to the rule described in Section 4.2.2 using the appropriate rebalancing boundary.

4.5.3 Numerical results from the Black-Scholes environment

In this section, we present the results of our numerical simulations in the Black-Scholes market.

Admissible set of rebalancing boundaries

Table 4.2 presents the average volatility deviations over 10,000 simulated risky asset paths of Portfolio S and Portfolio E for every rebalancing boundary $\phi \in A$. Column 2 (Portfolio S) reports the mean volatility deviation of Portfolio S (D^S). Column 3 to 7 report the percentage of the mean volatility deviation of Portfolio E (for each $\phi \in B$) in relation to Portfolio S. Recall (see (4.14)) that a rebalancing boundary level ϕ is admissible if the volatility deviation $D^{E,\phi}$ of Portfolio E does not exceed the benchmark Portfolio S volatility deviation D^S. Thus, when the annual market volatility $\sigma = 0.20$, all rebalancing boundary levels $\phi \in A$ are admissible. As the annual market volatility level increases the set B of admissible rebalancing boundaries reduces as follows: $B = \{0.005, 0.01, 0.015, 0.02\}$ for $\sigma = 0.30$, and $B = \{0.005, 0.01, 0.015\}$ for $\sigma = 0.40$ respectively.

Table 4.2: Average volatility deviations of Portfolio S and Portfolio E.

In this table, we report the mean volatility deviation of portfolio S, D^S, in column (2). The mean volatility deviations of portfolio E are reported as percentages of D^S from column (3) to column (7).

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Portfolio S	Portfolio E				
$\phi, \%$		0.50	1.00	1.50	2.00	2.50
$\sigma = 0.20$						
D^S and $\frac{D^{E,\phi}}{D^S}$, %	0.0042	33.0995	20.4517	20.0142	81.3817	76.2245
$\sigma = 0.30$						
D^S and $\frac{D^{E,\phi}}{D^S}$, %	0.0068	52.9412	48.5294	69.1176	91.1765	102.9412
$\sigma = 0.40$						
D^S and $\frac{D^{E,\phi}}{D^S}$, %	0.0086	59.3023	89.5349	93.0233	136.0465	139.5349

Omega ratios and optimal rebalancing boundaries

Table 4.3, Table 4.4, and Table 4.5 present the Omega ratios of Portfolio S and Portfolio E for the corresponding rebalancing boundary levels $\phi \in B$ when the annual market volatility $\sigma = 0.20, 0.30$, and 0.40, respectively.

Overall, we find that the optimal rebalancing boundary levels ϕ^* for Portfolio E are $\phi^* =$

Table 4.3: Omega ratios of Portfolio S and Portfolio E ($\phi \in B$). The minimum acceptable return for calculating the Omega ratio $a = 0, 0.01, 0.02, 0.03$, and 0.04. Annual market volatility $\sigma = 0.20$.

ϕ, %	Portfolio S	Portfolio E 0.50	1.00	1.50	2.00	2.50
$a = 0.00$						
Omega ratio, %	104.7384	104.7496	104.7516	104.7532	104.7538	104.7561
$a = 0.01$						
Omega ratio, %	103.3772	103.3879	103.3900	103.3913	103.3918	103.3936
$a = 0.02$						
Omega ratio, %	102.0468	102.0570	102.0592	102.0602	102.0607	102.0619
$a = 0.03$						
Omega ratio, %	100.7463	100.7560	100.7583	100.7589	100.7595	100.7602
$a = 0.04$						
Omega ratio, %	99.4746	99.4838	99.4861	99.4866	99.4871	99.4871

0.025 for $\sigma = 0.20$, $\phi^* = 0.020$ for $\sigma = 0.30$, and $\phi^* = 0.015$ for $\sigma = 0.40$. This is not surprising since a lower rebalancing boundary level allows for more frequent asset allocation adjustment, and therefore, better risk control in volatile market condition.

Further risk-return measurements

Table 4.6, Table 4.7, and Table 4.8 present further risk-return measurement comparison between Portfolio S and optimized Portfolio E under different market volatility conditions $\sigma = 0.20, 0.30$, and 0.40.

Table 4.6: Risk-return measurement: Portfolio S vs Optimized Portfolio E ($\phi^* = 0.025$) when the annual market volatility $\sigma = 0.20$

	Portfolio S	Optimized Portfolio E ($\phi^* = 0.025$)	Diff. (Optimized Portfolio E - Portfolio S)
Average annual return, %	3.5515	3.5636	0.0121
Average annualized volatility, %	12.0213	12.0256	0.0043
Sharpe ratio, %	12.9065	13.0020	0.0955
90% VAR, %	-11.8598	-11.8533	0.0065
95% VAR, %	-20.0102	-20.0067	0.0035
99% VAR, %	-24.4100	-24.4081	0.0019
Accumulated transaction cost (per year), $	1.5237	0.9551	-0.5686

Table 4.4: Omega ratios of Portfolio S and Portfolio E ($\phi \in B$). The minimum acceptable return for calculating the Omega ratio $a = 0, 0.01, 0.02, 0.03$, and 0.04. Annual market volatility $\sigma = 0.30$.

ϕ, %	Portfolio S	Portfolio E 0.50	1.00	1.50	2.00
$a = 0.00$					
Omega ratio, %	104.7384	104.7496	104.7516	104.7532	104.7538
$a = 0.01$					
Omega ratio, %	103.3772	103.3879	103.3900	103.3913	103.3918
$a = 0.02$					
Omega ratio, %	102.0468	102.0570	102.0592	102.0602	102.0607
$a = 0.03$					
Omega ratio, %	100.7463	100.7560	100.7583	100.7589	100.7595
$a = 0.04$					
Omega ratio, %	99.4746	99.4838	99.4861	99.4866	99.4871

Table 4.7: Risk-return measurement: Portfolio S vs Optimized Portfolio E ($\phi^* = 0.02$) when the annual market volatility $\sigma = 0.30$

	Portfolio S	Optimized Portfolio E ($\phi^* = 0.02$)	Diff. (Optimized Portfolio E - Portfolio S)
Average annual return, %	4.0652	4.0786	0.0134
Average annualized volatility, %	12.0192	12.0195	0.0003
Sharpe ratio, %	17.1822	17.2893	0.1071
90% VAR, %	-11.3435	-11.3304	0.0131
95% VAR, %	-19.4925	-19.4797	0.0128
99% VAR, %	-23.8916	-23.8788	0.0128
Accumulated transaction cost (per year), $	1.1285	0.7348	-0.3937

Table 4.8: Risk-return measurement: Portfolio S vs Optimized Portfolio E ($\phi^* = 0.015$) when the annual market volatility $\sigma = 0.40$

	Portfolio S	Optimized Portfolio E ($\phi^* = 0.015$)	Diff. (Optimized Portfolio E - Portfolio S)
Average annual return, %	3.5515	3.5615	0.0100
Average annualized volatility, %	12.0213	12.0253	0.0040
Sharpe ratio, %	12.9065	12.9895	0.0830
90% VAR, %	-11.8598	-11.8501	0.0097
95% VAR, %	-20.0102	-20.0001	0.0101
99% VAR, %	-24.4100	-24.0067	0.4033
Accumulated transaction cost (per year), $	0.9035	0.6082	-0.2953

Table 4.5: Omega ratios of Portfolio S and Portfolio E ($\phi \in B$). The minimum acceptable return for calculating the Omega ratio $a = 0, 0.01, 0.02, 0.03$, and 0.04. Annual market volatility $\sigma = 0.40$.

	Portfolio S	Portfolio E		
ϕ, %		0.50	1.00	1.50
$a = 0.00$				
Omega ratio, %	105.4222	105.4361	105.4385	105.4405
$a = 0.01$				
Omega ratio, %	104.0520	104.0655	104.0677	104.0696
$a = 0.02$				
Omega ratio, %	102.7129	102.7258	102.7279	102.7299
$a = 0.03$				
Omega ratio, %	101.4038	101.4163	101.4182	101.4201
$a = 0.04$				
Omega ratio, %	100.1238	100.1357	100.1375	100.1394

For all the market volatility scenarios considered, the optimized Portfolio E shows a higher average annual return, average annualized volatility, Sharpe ratio, 90%, 95%, and 99% VAR. When considering an initial investment of \$100, in contrast with Portfolio S, the optimized Portfolio E with rebalancing boundaries show a significantly lower average accumulated transaction cost (between \$0.2953 (when $\sigma = 0.40$) and \$0.5686 (when $\sigma = 0.20$) lower, in specific) on simulated paths.

One important assumption embedded in the Black-Scholes model is that market returns are normally distributed. However, it is widely conceived that this normality assumption is controversial. For example, Fama (1965) reveals that stock returns conform better to the stable Paretian distribution. Kon (1984) posits that a mixture of discrete normal distribution is more suitable to describe daily stock returns due to their significant positive skewness.

4.6 Numerical optimization based on real market data

In this section, we demonstrate the performance of our numerical optimization algorithm using real market data. Suppose on Dec 31st, 2010, an investor is looking to set up his/her

Target Volatility Portfolio E based on S&P 500 as a risky asset and bond (with a constant growing rate of $r = 2\%$) as a risk-free asset and an initial investment of \$100. The investor would like to apply our numerical optimization algorithm.

4.6.1 Bootstrap risky asset paths simulation

Step one of our numerical optimization algorithm described in section 4.4.2 consists of simulating ”future” risky asset paths. Suppose an investor chooses the moving block bootstrap method, or the so-called overlapping bootstrap method for the risky asset paths simulation.

Remark. This method was first proposed by Kunsch (1989) and Liu and Singh (1992). The bootstrap method does not impose any assumptions on the distribution of market returns (Ruiz and Pascual, 2002). In addition, comparing to other bootstrapping approaches, the moving block bootstrap method is shown to be more reliable in preserving the characteristics of stock returns such as auto-correlation, fat-tail, and positive skewness (Lahiri, 1999; Radovanov and Marcikić, 2014; Pažickỳ et al., 2017).

On Dec 31st, 2010, an investor would like to use the moving block bootstrap method to simulate 10,000 risky asset return paths for a 10-year future investment period. The simulation requires the following steps:

1. Download daily returns of S&P 500 over a recent historical period. For our numerical experiments we have used the time period from Dec 31st, 1980 to Dec 31st, 2010 (30-year worth of data prior to Dec 31st, 2010).

2. For each simulation, generate one risky-asset return path that includes 10 blocks of returns. Each block contains 252 consecutive daily returns from a randomly selected starting date in the S&P 500 time series data. This gives us a 10-year long, risky asset return paths;

3. Repeat the simulation 10,000 times and result in 10,000 risky asset return paths.

4.6.2 Choosing an initial optimal rebalancing boundary

On Dec 31st, 2010, an investor runs our optimization algorithms using the 10,000 bootstrap simulated "future" S&P 500 paths with the 10-year investment horizon (from Dec 31st, 2010 to Dec 31st, 2020).

As presented in Table 4.9, in the first stage of the algorithm, based on the constraint criteria of the mean volatility deviation of Portfolio S and Portfolio E ($D^{E,\phi} \leq D^S$), the set B of admissible rebalancing boundaries is determined. It consists of the values $\phi = 0.005$, and 0.010. In the second stage of the algorithm, an optimal value $\phi^* = 0.010$ is determined based on the Omega ratio comparison. The investor would use the optimal rebalancing boundary $\phi^* = 0.010$ for Portfolio E starting from the beginning of the 10-year future investment horizon.

Table 4.9: Mean volatility deviations and Omega ratios of Portfolio S and Portfolio E (with $\phi \in B$). Data for bootstrapping: S&P 500, Dec 31st, 1980 - Dec 31st, 2010. $r = 0.02$.

	Portfolio S	Portfolio E				
ϕ, %		0.50	1.00	1.50	2.00	2.50
Mean volatility deviations, %	1.1049	1.1012	1.1049	1.1160	1.1212	1.1312
$a = 0.00$ Omega ratio, %	88.1091	88.1786	88.2507			
$a = 0.01$ Omega ratio, %	86.9476	87.0152	87.0859			
$a = 0.02$ Omega ratio, %	85.8142	85.8799	85.9494			
$a = 0.03$ Omega ratio, %	84.7075	84.7715	84.8398			
$a = 0.04$ Omega ratio, %	83.6265	83.6888	83.7560			

4.6.3 Risk calibrated rebalancing boundary levels

In Section 4.6.2, we demonstrated how an initial optimal rebalancing boundary $\phi^* = 0.010$ has been chosen. Suppose the investor's Portfolio E with the initial optimal rebalancing boundary $\phi^* = 0.010$ runs for a certain period of time within the 10-year investment horizon. Depending on the market situation, an investor may choose to recalibrate his/her choice of the rebalancing boundary. For this study, we demonstrate how such recalbration may be performed 5 years into the investment horizon, on Dec 31st, 2015.

We have used the actual S&P 500 path from Dec 31st, 2010 (when our hypothetical Portfolio E has been set up) to Dec 31st, 2015 to evaluate the performance of Portfolio E (with $\phi^* = 0.010$) in comparison with Portfolio S as of Dec 31st, 2015. Table 4.10 presents the risk-return measurements for the two portfolios assuming that the investor invested $100 initially on Dec 31st, 2010.

Table 4.10: Risk-return measurement: Portfolio S vs Optimized Portfolio E ($\phi^* = 0.010$). Investment horizon: Dec 31st, 2010 - Dec 31st, 2015.

	Portfolio S	Optimized Portfolio E ($\phi^* = 0.010$)	Diff. (Optimized Portfolio E - Portfolio S)
Average annual return, %	6.4744	6.5682	0.0938
Average annualized volatility, %	12.9133	12.9165	0.0032
Sharpe ratio, %	0.8361	0.8484	0.0123
90% VAR, %	-15.3130	-15.2209	0.0922
95% VAR, %	-19.3143	-19.2231	0.0912
99% VAR, %	-24.0900	-23.9999	0.0901
Accumulated transaction cost (per year), $	1.8688	1.8476	-0.0529

Based on the risk-return measurements on real historical data, in contrast with Portfolio S, the Optimized Portfolio E shows a $0.0529 lower accumulated transaction cost per year. Meanwhile, the Optimized Portfolio E also has a 0.0922, 0.0912, and 0.0901% higher 90%, 95%, and 99% VAR compared to Portfolio S, suggesting that the Optimized Portfolio E can better control portfolio downside risk than Portfolio S. With respect to other performance measures, the Optimized Portfolio E results in a 0.0938 higher average annual return when comparing to its counterpart Portfolio S. In addition, we also see that the Optimized Portfolio E shows a higher average annualized volatility. This is not surprising, since adding the rebalancing boundary level reduces the frequency of risky and risk-free asset allocation adjustment as an attempt to decrease transaction cost. In addition, the Optimized Portfolio E shows a 0.0123% higher Sharpe ratio in contrast with Portfolio S, indicating that the Optimized Portfolio E earns a higher return when looking at the same level of risk.

On Dec 31st, 2015, the investor runs our numerical optimization algorithm again using the following input data:

- The latest 30-year worth of data of the S&P 500 data from Dec 31st, 1985 to Dec 31st, 2015;
- The future investment horizon of 5 years;

Results regarding the mean volatility deviation and Omega ratio from the rebalancing boundary recalibration are shown in Table 4.11. In the first stage of the algorithm, the set B of admissible rebalancing boundaries is determined. It consists of the value $\phi = 0.005$. Thus, from Dec 31st, 2015 forward, the investor would adjust the initially selected rebalancing boundary level $\phi^* = 0.010$ to $\phi^* = 0.005$ for Portfolio E.

4.6.4 Numerical results

We have used the actual S&P 500 path from Dec 31st, 2010 (when our hypothetical Portfolio E has been set up) through the end of the 10-year investment period (Dec 31st, 2020) to evaluate the performance of Portfolio E. Noticed that the Portfolio E started with an optimal rebalancing boundary $\phi^* = 0.010$ on Dec 31st, 2010. This rebalancing boundary level was recalibrated to $\phi^* = 0.005$ on Dec 31st, 2015 based on the re-implementation of our numerical optimization algorithm. Table 4.12 presents the risk-return measurements

Table 4.11: Mean volatility deviations and Omega ratios of Portfolio S and Portfolio E (with $\phi \in B$). Data for bootstrapping: S&P 500, Dec 31st, 1985 - Dec 31st, 2015. $r = 0.02$.

	Portfolio S	Portfolio E				
ϕ, %		0.50	1.00	1.50	2.00	2.50
Mean volatility deviations, %	1.0700	1.0645	1.0733	1.0799	1.0845	1.0924
$a = 0.00$ Omega ratio, %	133.7524	133.8208				
$a = 0.01$ Omega ratio, %	131.8694	131.9351				
$a = 0.02$ Omega ratio, %	130.0305	130.0932				
$a = 0.03$ Omega ratio, %	128.2359	128.2958				
$a = 0.04$ Omega ratio, %	126.4836	126.5407				

for Portfolio S and Portfolio E (with risk calibrated rebalancing boundary levels).

Table 4.12: Risk-return measurement: Portfolio S vs Optimized Portfolio E ($\phi^* = 0.010$ and 0.005). Investment horizon: S&P 500, Dec 31st, 2010 - Dec 31st, 2020.

	Portfolio S	Optimized Portfolio E ($\phi^* = 0.010$ and 0.005)	Diff. (Optimized Portfolio E - Portfolio S)
Average annual return, %	9.9003	9.9688	0.0685
Average annualized volatility, %	12.8076	12.8095	0.0019
Sharpe ratio, %	1.2680	1.2770	0.0090
90% VAR, %	-11.8503	-11.7809	0.0694
95% VAR, %	-15.8336	-15.7647	0.0688
99% VAR, %	-20.5878	-20.5197	0.0681
Accumulated transaction cost (per year), $	1.5902	1.5371	-0.0531

When looking at the 10-year investment horizon (Dec 31st, 2010 to Dec 31st, 2020), with the risk calibrated rebalancing boundary levels, the Optimized Portfolio E has a \$0.0531 lower accumulated transaction cost in contrast with Portfolio S. Regarding the portfolio return measurements, compared to Portfolio S, the Optimized Portfolio E results in a 0.0685% higher average annual return and a 0.0090% higher Sharpe ratio. Similar to the results reported in Table 4.11 where the rebalancing boundary level $\phi^* = 0.010$ was used, we see that the Optimized Portfolio E has a 0.0019% higher average annualized volatility. Nevertheless, the higher 90%, 95%, and 99% of the Optimized Portfolio E suggest that it is more capable of controlling portfolio downside risk when comparing with Portfolio S.

In summary, we find the most suitable rebalancing boundary level by implementing our numerical optimization algorithm on bootstrapped risky asset paths from real market data. The optimal rebalancing boundary level, based on our analysis, may change when looking at different investment horizons. Our risk-return measurements over two fixed investment periods (Dec 31st, 2010 - Dec 31st, 2015 & Dec 31st, 2010 - Dec 31st, 2020) suggest that the Optimized Portfolio E, with risk calibrated rebalancing boundary levels can reduce the accumulated transaction cost of a target volatility portfolio and increase portfolio return.

More importantly, we also find that applying rebalancing boundary to the volatility target asset allocation mechanism does not reduce the risk-control capability of the target volatility portfolio, which is the core of the target volatility investment concept. As shown in Table 4.11 and Table 4.12, the Optimized Portfolio E shows a higher 90%, 95%, and 99% VAR in contrast with Portfolio S. We also confirm this pattern by looking at five different market scenarios (see section 4.7). This can be attributed to our optimization algorithm design, which only considers admissible rebalancing boundary levels ϕ such that the mean volatility deviation of portfolio E does not go beyond the benchmark volatility deviation ($D^{E,\phi} \leq D^{S}$).

4.7 Analysis in different market environments

In this section, we provide a comparative analysis of performance of Portfolio E with fixed rebalancing boundary versus the benchmark Portfolio S in different market environments using historical market data. We follow Biedova and Steblovskaya (2020) and look at four investment horizons that reflect different market conditions. In addition, we include the

2020 - 2021 investment horizon which contains the most recent market drawdown due to the COVID-19 pandemic. Illustrations of the five investment horizons are shown in Figure 4.2. For this analysis, we fix a rebalancing boundary for Portfolio E at $\phi = 0.010$ since each investment horizon is 2-year long only. Similar to our analysis using the bootstrap method, we also assume that \$100 was initially invested at the beginning of each investment horizon.

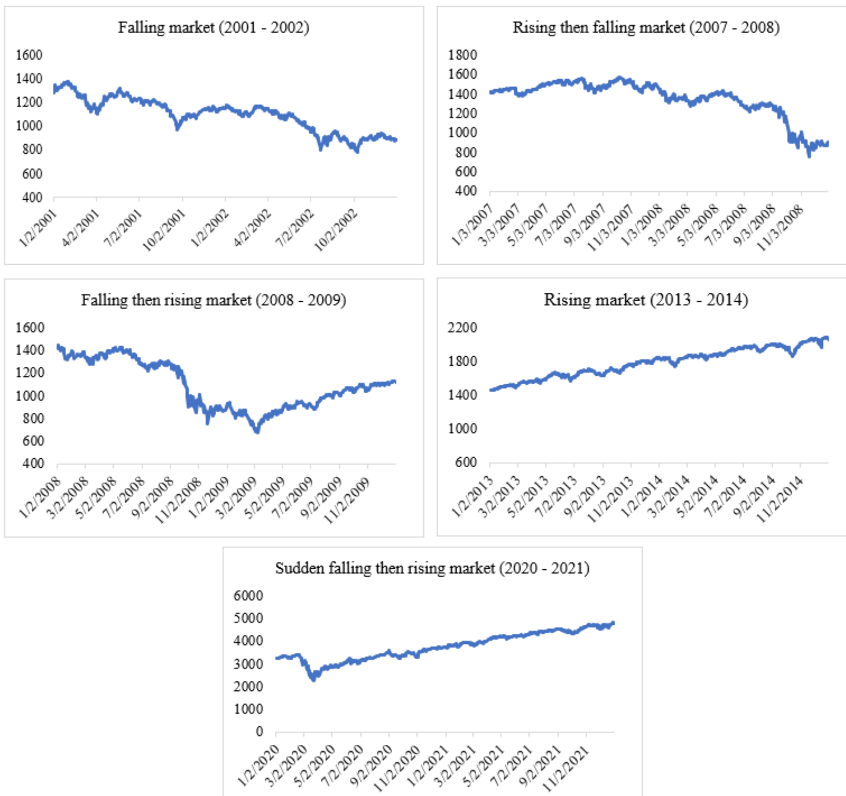


Figure 4.2: Investment horizons for historical analysis.

We download data from the Bloomberg database and the Federal Reserve Economic Data database. In Table 4.12, we present the risk-return measurements for Portfolio S and Portfolio E over the four investment horizons.

For all the five investment horizons, in contrast with the target volatility portfolio without the rebalancing boundary (Portfolio S), the Portfolio E with $\phi = 0.010$ shows higher average annual return, Sharpe ratio, and 90%, 95%, and 99% VAR. Regarding the transaction cost, the Portfolio E also has a lower accumulated transaction cost (per year) compared to

Table 4.13: Risk-return measurement: Portfolio S vs Portfolio E ($\phi = 0.010$) on historical market data.

	Portfolio S	Portfolio E ($\phi = 0.010$)	Diff. (Portfolio E - Portfolio S)
2001 - 2002			
Average annual return, %	-14.2858	-14.2086	0.0773
Average annualized volatility, %	12.5528	12.543	0.0015
Adjusted Sharpe ratio, %	-1.7933	-1.7838	0.0015
90% VAR, %	-34.9720	-34.8973	0.0095
95% VAR, %	-38.8593	-38.7851	0.0743
99% VAR, %	-43.4990	-43.4253	0.0737
Accumulated transaction cost, $	2.3838	2.2400	-0.1438
2007 - 2008			
Average annual return, %	-8.1115	-7.9162	0.1953
Average annualized volatility, %	13.0651	13.0624	-0.0027
Sharpe ratio, %	-1.0598	-1.0340	0.0257
90% VAR, %	-30.0538	-29.8490	0.2048
95% VAR, %	-34.1041	-33.8984	0.2057
99% VAR, %	-38.9383	-38.7317	0.2067
Accumulated transaction cost, $	2.7942	2.7390	-0.1126
2008 - 2009			
Average annual return, %	-2.1184	-1.8917	0.2267
Average annualized volatility, %	12.2492	12.2382	-0.0109
Sharpe ratio, %	-0.2595	-0.2315	0.0280
90% VAR, %	-23.4604	-23.2114	0.2490
95% VAR, %	-27.2690	-27.0165	0.2524
99% VAR, %	-31.8147	-31.5582	0.2565
Accumulated transaction cost, $	2.1849	2.0393	-0.1456
2013 - 2014			
Average annual return, %	11.3544	11.4696	0.1152
Average annualized volatility, %	12.8835	12.8841	0.0006
Sharpe ratio, %	88.1313	89.0213	0.8900
90% VAR, %	-10.3172	-10.2027	0.1145
95% VAR, %	-14.3109	-14.1996	0.1143
99% VAR, %	-19.0776	-18.9635	0.1141
Accumulated transaction cost, $	3.9611	3.8671	-0.0940
2020 - 2021			
Average annual return, %	15.6346	15.6929	0.0582
Average annualized volatility, %	13.8023	13.8089	0.0066
Sharpe ratio, %	2.1579	2.1670	0.0091
90% VAR, %	-7.1396	-7.0930	0.0467
95% VAR, %	-11.4172	-11.3726	0.0446
99% VAR, %	-16.5227	-16.4805	0.0446
Accumulated transaction cost, $	1.7481	1.7258	-0.0543

Portfolio *S*.

Despite this board tendency, regarding the ability of portfolio risk control, which is the center of the target volatility investment concept, the Optimized portfolio *E* performs the best when looking at the investment horizon 2008 - 2009. Specifically, the Optimized Portfolio *E* not only shows a 0.2267% higher average annual return but also a 0.0109% lower annualized volatility in contrast with Portfolio *S*. Meanwhile, we also see the most pronounced difference between Portfolio *E* and Portfolio *S* regarding the 90%, 95%, and 99% VAR. This provides evidence that adding a suitable rebalancing boundary level to an existing volatility target asset allocation mechanism my further increase its risk control capability when the portfolio faces a market turbulent event. Regarding the transaction cost, this investment period shows the most significant transaction cost reduction ($0.1456) when comparing to other investment horizons.

4.8 Conclusions and future directions

In this paper, we attempt to reduce the transaction cost of a target volatility portfolio by including rebalancing boundaries to its volatility target asset allocation mechanism. A constraint optimization problem is formulated to find the optimal rebalancing boundary over a certain period of investment horizon. We implement our constraint optimization algorithm in an artificial Black-Scholes environment as well as using bootstrap method on real market data. From our analysis using the bootstrap method on market data, we find that the extended target volatility portfolio with a suitable selection of rebalancing boundary level could reduce the accumulated transaction cost and improve portfolio return.

In addition, we also showed that an investor may recalibrate the rebalancing boundary based on changing portfolio risk and the optimal value of rebalancing boundary may vary during an investment horizon. Following our findings, in pension practice and future work, we might consider applying asymmetrical, volatility-linked rebalancing boundary levels linked to the different levels of risky asset allocation of the target volatility portfolio. This setup allows as to further control the risky and risk-free asset rebalancing mechanism with regard to portfolio risk and specific market conditions. In addition, one may also consider using non-linear rebalancing boundary levels under which we apply rebalance boundaries to the volatility target asset allocation mechanism when the risky asset allocation needs to be increased. Under this design, the target volatility portfolio can quickly reduce its risky asset allocation when market volatility is high and may further protect the portfolio from overly exposing to risky asset when market volatility drops down.

Appendix A

This appendix includes the comprehensive results for our simulation analysis when the initial annual interest rate $r(0) = 0\%$ in Chapter 3.

Table 14: The length of the retirement coverage (in years). Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP), equities/bonds, %					
30/70	29.90	29.89	29.88	29.87	29.85
40/60	29.62	29.60	29.57	29.53	29.49
50/50	29.17	29.13	29.08	29.01	28.93
60/40	28.58	28.53	28.46	28.37	38.25
70/30	29.40	29.46	29.53	29.59	29.65
Target volatility portfolio (TVP), equities/bonds, %					
30/70	30.00	30.00	30.00	30.00	30.00
40/60	30.00	30.00	30.00	30.00	30.00
50/50	29.99	29.99	29.99	30.00	30.00
60/40	29.85	29.88	29.90	29.92	29.93
70/30	29.40	29.46	29.53	29.59	29.65
Difference (TVP - BP), equities/bonds, %					
30/70	0.10	0.11	0.12	0.13	0.15
40/60	0.38	0.40	0.43	0.47	0.51
50/50	0.82	0.86	0.91	0.99	1.07
60/40	1.27	1.34	1.44	1.55	1.68
70/30	1.47	1.60	1.76	1.94	2.14

Table 15: Portfolio survival rate, %. Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP), equities/bonds, %					
30/70	96.49	96.41	96.19	95.78	95.38
40/60	90.61	90.20	89.70	89.15	88.45
50/50	83.22	82.81	82.08	81.34	80.57
60/40	76.41	75.92	75.36	74.63	73.74
70/30	70.02	69.56	69.13	68.50	67.47
Target volatility portfolio (TVP), equities/bonds, %					
30/70	100.00	100.00	100.00	100.00	100.00
40/60	99.99	100.00	100.00	100.00	100.00
50/50	99.41	99.54	99.68	99.83	99.89
60/40	95.24	95.85	96.54	96.97	97.50
70/30	86.78	87.69	88.76	89.98	90.96
Difference (TVP - BP), equities/bonds, %					
30/70	3.51	3.59	3.81	4.22	4.62
40/60	9.38	9.80	10.30	10.85	11.55
50/50	16.19	16.73	17.60	18.49	19.32
60/40	18.83	19.93	21.18	22.34	23.76
70/30	16.76	18.13	19.63	21.48	23.49

Table 16: Portfolio 95% VAR (in thousand dollars). Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP), equities/bonds, %					
30/70	43.58	35.30	27.29	19.45	9.28
40/60	-98.27	-105.44	-114.74	-125.94	-140.69
50/50	-213.98	-226.45	-236.64	-249.94	-265.42
60/40	-319.69	-328.09	-340.64	-357.17	-374.60
70/30	-411.88	-419.46	-436.80	-456.47	-475.27
Target volatility portfolio (TVP), equities/bonds, %					
30/70	415.56	419.46	422.05	425.62	428.07
40/60	320.10	327.71	335.50	341.59	346.76
50/50	174.41	186.41	200.55	211.83	223.48
60/40	6.75	21.71	39.51	56.34	73.65
70/30	-161.54	-146.72	-128.44	-110.06	-90.39
Difference (TVP - BP), equities/bonds, %					
30/70	371.98	384.16	394.75	406.17	418.79
40/60	418.37	433.15	450.24	467.54	487.45
50/50	388.39	412.86	437.19	461.77	488.90
60/40	326.44	349.80	380.15	413.52	448.25
70/30	250.34	272.74	308.36	346.41	384.88

Table 17: Portfolio 95% CVAR (in thousand dollars). Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP), equities/bonds, %					
30/70	-53.43	-59.11	-67.04	-77.72	-90.62
40/60	-200.82	-208.05	-218.26	-231.67	-248.00
50/50	-329.46	-338.63	-351.57	-368.10	-387.87
60/40	-448.37	-459.96	-475.51	-494.70	-518.15
70/30	-567.32	-580.90	-599.68	-623.59	-651.30
Target volatility portfolio (TVP), equities/bonds, %					
30/7	363.11	366.93	370.35	373.53	376.49
40/60	252.64	261.09	269.47	277.48	285.06
50/50	93.05	105.83	119.83	133.03	145.96
60/40	-87.76	-71.60	-53.25	-34.76	-16.97
70/30	-270.59	-252.26	-230.31	-207.56	-185.34
Difference (TVP - BP), equities/bonds, %					
30/70	416.54	426.05	437.39	451.25	467.11
40/60	453.46	469.14	187.73	509.15	533.06
50/50	422.52	444.45	471.10	501.13	533.83
60/40	360.61	388.36	422.27	459.94	501.18
70/30	296.73	328.64	369.37	416.03	465.97

Table 18: Average annualized return, %. Initial annual interest rate $r(0) = 0\%$

$\sigma(0), \%$	10	15	20	25	30
Benchmark portfolio (BP), equities/bonds, %					
30/70	4.329	4.329	4.329	4.330	4.330
40/60	4.346	4.347	4.347	4.348	4.348
50/50	4.365	4.365	4.366	4.367	4.367
60/40	4.384	4.386	4.387	4.387	4.388
70/30	4.405	4.407	4.408	4.409	4.410
Target volatility portfolio (TVP), equities/bonds, %					
30/70	4.269	4.274	4.277	4.279	4.281
40/60	4.269	4.275	4.279	4.281	4.283
50/50	4.277	4.280	4.284	4.287	4.289
60/40	4.296	4.291	4.294	4.297	4.299
70/30	4.332	4.312	4.310	4.312	4.314
Difference (TVP - BP), equities/bonds, %					
30/70	-0.06	-0.055	-0.052	-0.050	-0.049
40/60	-0.077	-0.072	-0.068	-0.066	-0.065
50/50	-0.088	-0.086	-0.082	-0.080	-0.078
60/40	-0.089	-0.095	-0.092	-0.090	-0.089
70/30	-0.073	-0.095	-0.098	-0.097	-0.096

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This manuscript was typed by the author.